Core Course – 9

Elements of Modern Physics

Problem Set – 1 (Unit 1)

Corpuscular Nature of Radiation

1. Find the number of photons emitted from a monochromatic bulb of power 1 W at wavelength 500 nm.

2. The wavelength corresponding to maximum energy of solar radiation is 482 nm. Estimate the temperature of the sun.

3. The wavelength corresponding to maximum radiation from human body is 10 μm while that from the sun is 482 nm. Use these data to calculate the temperature of the sun.

4. Find the energy of a photon of wavelength 1 A° in electron-volt unit. What is its momentum? Compare it with the momentum of an electron moving with the kinetic energy same as the energy of this photon.

5. A body absorbs 10^{20} photons per sec. Find the average force exerted on the body. Given, the wavelength of radiation is 500 nm.

Photoelectric Effect

6. The photoelectric work function of sodium is 2.3 eV. Find the threshold frequency.

7. Monochromatic radiation at wavelength 300 nm is incident on a piece of barium (photoelectric work function 2.5 eV). Will there be any photoelectric emission? If yes, find the maximum kinetic energy of the photoelectron. Also find maximum velocity of the photoelectrons. What is the stopping potential in this case?

8. When light of wavelength 450 nm and 300 nm are incident on a photocathode, the emitted photoelectrons are found to have stopping potential 0.5 V and 2 V respectively. Calculate the value of Planck's constant from these data.

Compton Effect

9. Show that in Compton scattering while the photon is scattered at angles between 0° to 180°, the recoil electron is emitted at angles between 0° to 90°.

10. X-rays with wavelength 1 A° are scattered from a carbon block. The scattered radiation is viewed at 90° to the incident beam. Calculate the Compton shift and the kinetic energy imparted to the recoiled electron.

11. Show that the maximum kinetic energy of the recoiled electron in Compton scattering is given by

$$T_{max} = \frac{2h^2\upsilon^2/m_0c^2}{(1+2h\upsilon/m_0c^2)},$$

where the symbols have their usual meanings.

12. Show that the Compton shift, under non-relativistic case (kinetic energy of the recoiled electron is very small), is given by

$$\Delta\lambda = \lambda' - \lambda = \frac{\lambda_c}{2}\left(\frac{\lambda'}{\lambda} + \frac{\lambda}{\lambda'} - 2\cos\phi\right),$$

where ϕ is the photon scattering angle, and other symbols have their usual significances. [S N Ghoshal]

13. Show that it is impossible to transfer all the energy of the photon to the electron.

Matter Wave

14. A stone of mass 100 g is dropped from the top of a building of height 50 m. Find the de Broglie wavelength when the stone reaches the ground.

15. Find the energy and momentum of an electron having de Broglie wavelength 1 Å.

16. Show that the relativistic de Broglie wavelength is given by

$$\lambda_r = \frac{hc}{\sqrt{E_k(E_k + 2m_0c^2)}} ,$$

where the symbols have their useful meanings.

17. An electron and a proton has same de Broglie wavelength. Prove that the energy of the electron is greater.

18. Consider the nth orbit of a hydrogen like atom to fit to n number of stationary de Broglie wave. Show that it leads to Bohr's postulate of quantization of angular momentum.

19. The kinetic energy of an electron is equal to its rest mass energy. Find the de Broglie wavelength associated with it in terms of the Planck constant (h), rest mass (m_0) and speed of light in free space (c).

20. An electron has de Broglie wavelength 2 pm. Find the phase velocity and the group velocity of its de Broglie wave. Given the rest mass energy of the electron is 511 keV.

21. A particle of charge e and rest mass m_0 is accelerated by an electric potential V. What is the de Broglie wavelength of the particle viewing the relativistic energy momentum relation?

22. An electron is accelerated by a potential 40 kV. Find the de Broglie wavelength associated with it considering (i) the relativistic effect and (ii) neglecting relativistic effect.

23. In Davison-Germer experiment, the diffraction maxima occurs at an angle $50°$ when the electron accelerating potential is 54 V and the atomic spacing for single nickel crystal is 2.15 Å. From this data show that the experiment is in agreement with de Broglie hypothesis.

24. Calculate the de Broglie wavelength of free electron gas at 0 ℃. Assume free electrons as ideal gas.

25. Assuming the relation between the phase velocity and group velocity, prove that for low energy electrons, the group velocity is equal to the velocity of the electrons. Use non-relativistic expression. [S N Ghoshal]

26. The de Broglie wavelength of an electron is 0.15 Å. Find the phase velocity and group velocity of the de Broglie waves. Also find the kinetic energy of the electrons. Given, the rest mass energy of the electron is 0.511 MeV/c^2.

Heisenberg's Uncertainty Principle

27. Use Heisenberg's uncertainty principle to prove that electron can't reside inside the nucleus.

28. Consider a particle confined in a one dimensional box of length l. Using Heisenberg's uncertainty principle estimate the minimum energy of the particle.

29. Use Heisenberg's uncertainty principle to estimate the Bohr's radius and hence the ground state energy of hydrogen atom.

30. Using Heisenberg's uncertainty principle estimate the minimum energy of a harmonic oscillator.

31. Assume that an electron is inside a nucleus of radius 10^{-15} m. Calculate from the uncertainty principle the minimum kinetic energy of the electron.

32. Compute the smallest possible uncertainty in the position of an electron moving with velocity 3×10^7 m/s. The rest mass energy of the electron is 9.1×10^{-31} kg.

33. An electron of energy 200 eV is passed through a circular hole of radius 10^{-4} cm. What is the uncertainty introduced in the angle of emergence? What would be the corresponding uncertainty for a 1 g lead ball thrown with a velocity 10^3 cm/sec through a hole 1 cm in radius?

34. The position and momentum of a 4.4 keV electron are simultaneously determined. If its position is located to within 1.1 Å, what is the percentage of uncertainty in its momentum?

35. An atomic state has a lifetime of 10 ns. Find the natural line-width of the corresponding electromagnetic radiation.

36. In a single slit diffraction experiment with monoenergetic electron beam of 100 keV a slit of width 1 μm is used. Using uncertainty principle, estimate the angle at which the first order diffraction minima appears.

37. A meson has lifetime 10^{-22} sec. To what accuracy can its rest mass be known?

Review of CU Exam. Papers:

CU – 2019

1. Explain how a wave packet may be formed by the superposition of two monochromatic waves of slightly different wavelengths. Define group velocity and phase velocity. Find a relation between them. [4+(2+2)+2]

CU – 2018

1. Calculate the value of Compton wavelength of an electron. [2]

2. A single sinusoidal wave can't represent a localized particle – Explain. [2]

3. The photoelectric work-function W for Li is 2.3 eV. Find the threshold wavelength λ_0 for photoelectric effect. If ultraviolet light of $\lambda = 2000$ Å is incident on Li surface, obtain the maximum kinetic energy of photoelectrons and the value of stopping potential. [3]

4. Draw the polar graph of data obtained from Davisson-Germer experiment indicating clearly the coordinates used. Mention also the potential and angle at which maximum intensity is obtained. What information can we get from this experiment? [3]

CU – 2017

1. The human eye can detect 1.0×10^{-18} J of electromagnetic energy. How many 600 nm photons does this represent? [2]

2. If the uncertainty in the position of a particle be determined up to an accuracy of 10^{-8} m, what is the corresponding uncertainty in momentum? [2]

3. (a) Using the concept of discrete energy and Boltzmann probability distribution, find the average energy per mode of oscillation of an assembly of harmonic oscillators. Hence derive the spectral distribution function assuming the number of modes per unit volume per unit wavelength to be $(8\pi/\lambda^4)$. What is the name of the spectral distribution obtained above? [3+1]

(b) Repeat the above treatment for the case of continuous variation in energy. Mention the name of the spectral distribution obtained. [3+1]

4. A layer of sodium 1 atom thick and 1 m^2 in area contain 10^{19} atoms. If the incident light delivers power of 10^{-25} watts to each atom, then what would be the time required for an atom to accumulate sufficient energy to emit a photo-electron classically (Given work function for sodium = 2.3 eV)? What inference can be drawn regarding the Einstein's Photoelectric effect from the above problem? [2]

5. An electron and a proton have the same kinetic energy. Compare the wavelengths and the phase and group velocities of their de Broglie waves. [3]

CU – 2016

1. The kinetic energy of a thermal neutron of mass 1.675×10^{-27} kg is 3.35×10^{-21} joule. Calculate its de Broglie wavelength. [2]

2. Uncertainty principle forces us to reject Bohr's model of H-atom. Discuss. [2]

3. The uncertainty in the velocity of a particle is equal to its velocity. If $\Delta p_x \Delta x \approx h,$ show that uncertainty in its location is its de-Broglie wavelength. [2]

CU – 2015

1. Find the number of photons emitted per second by a 40 Watt source of light of wavelength 6000 Å. [2]

2. Show that the de Broglie wavelength of an electron is equal to its Compton wavelength when its speed is $c/\sqrt{2}$. [2]

3. An electron has a de Broglie wavelength of 0.15 Å. Compute the phase and group velocities of that wave. [3]

4. An electron of energy 200 eV is passed through a circular hole of radius 10^{-4} cm. What is the uncertainty introduced in the angle of emergence? [3]

CU – 2014

1. What is the minimum uncertainty in momentum of an electron confined in a space of linear dimension 1 Å? ($h - 6.67 \times 10^{-34}$ joule-sec) [2]

2. For a free particle, show that the group velocity of associated wave is equal to the classical velocity of the particle. [2]

3. What modification is introduced in Bohr's theory due to the finite mass of the nucleus? [2]

CU – 2012

1. The kinetic energy of a thermal neutron of mass 1.675×10^{-27} kg is 3.35×10^{-21} joule. Calculate its de Broglie wavelength. [2]

2. Calculate the Compton wavelength of an electron. [2]

3. If the uncertainty in the position of a particle be determined up to an accuracy 10^{-8} m, what is the corresponding uncertainty in momentum? Is the uncertainty in velocity negligible? [3]

CU – 2011

1. Calculate the de Broglie wavelength of corresponding to the Fermi momentum of a free electron gas at $T = 0$. [2]

2. A 2 MeV gamma photon is scattered through an angle of $180°$ by a free electron at rest. What is the recoil kinetic energy of the electron? [2]

3. Life-time of a nucleus in the excited state is 10^{-12} s. Calculate the probable uncertainty in energy and frequency of a γ ray photon emitted by it. [3]