DSC – 5: Modern Physics

***Syllabus:* (Radiation and Its Nature)**

Black body Radiation, Planck's quantum hypothesis, Planck's constant (derivation of Planck formula is not required). Photoelectric effect and Compton scattering — light as a collection of photons. Davisson-Germer experiment. Bohr-Sommerfeld quantization of the form $\oint p dq = nh$. De Broglie wavelength and matter waves. Wave-particle duality. Wave description of particles by wave packets. Group and Phase velocities and relation between them. Probability interpretation: Normalized wave functions as probability amplitudes. Two-slit experiment with photons and electrons. Linear superposition principle as a consequence. Position measurement, γ-ray microscope thought experiment. Heisenberg uncertainty principle (Statement with illustrations). Impossibility of a trajectory of a particle.

Modern Physics

References:

(1) Introductory Quantum Mechanics by S. N. Ghoshal – Calcutta Book House

(2) Quantum Mechanics by B.H. Bransden & C.J. Joachain – Pearson √

(3) Quantum Mechanics – Theory and Applications by A. Ghatak & S. Lokanathan – Trinity

(4) Quantum Mechanics by J. L. Powell & B. Crasemann – Addison Wesley

(5) Introduction to Quantum Mechanics by David J. Griffiths & Darrell F. Schroeter – Cambridge

(6) Problems in Modern Physics by P. Mandal – Techno World (2^{nd} Edition) √

Modern Physics

Chronology

- 1900: Max Planck – Theoretical explanation of blackbody radiation based on quantum hypothesis
- 1905: Albert Einstein – Explanation of photoelectric effect based on light quantum hypothesis
- 1913: Niels Bohr – Hydrogen atom model based on quantized angular momentum postulate
- 1914: James Frank & Gustav Ludwig Hertz – Experiment on electron collisions with mercury atoms providing a new test of Bohr's quantized model of atomic energy levels
- 1922: Otto Stern & Walther Gerlach – Deflection of neutral atomic beam by inhomogeneous magnetic field exhibiting space quantization of atomic magnetic moments
- 1923: Arthur Compton – Scattering of X-ray by free electrons (Compton scattering)

Modern Physics

Chronology

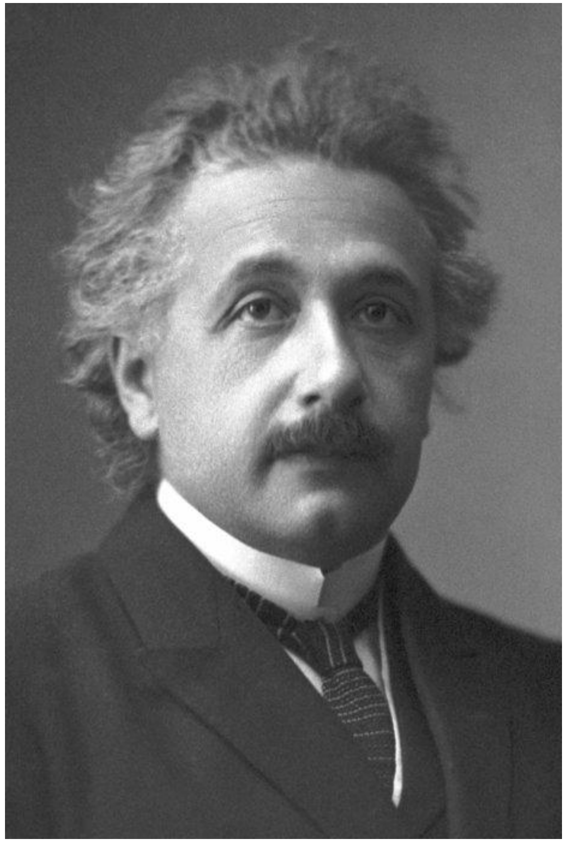
- 1924: Louis de Broglie – Wave-particle duality extended to particle (matter waves)
- 1925: Wolfgang Pauli – Exclusion principle for electrons
- 1925: Uhlenbeck & Goudsmit – Idea on intrinsic spin of electron
- 1925-1928: Heisenberg, Born, Jordan, Schrodinger, Dirac – Development of quantum mechanics
- 1927: Davisson & Germer, G. P. Thomson – Experimental discovery of the diffraction of electrons by crystals
- 1928: Gamow, Gurney & Condon – First application of quantum mechanics-penetration through potential barrier-explanation of alpha decay

Nobel Laureates: Quantum Mechanics

1919: "in recognition of the services he rendered to the advancement of Physics by his discovery of energy quanta"

Max Karl Ernst Ludwig Planck

Nobel Laureates: Quantum Mechanics



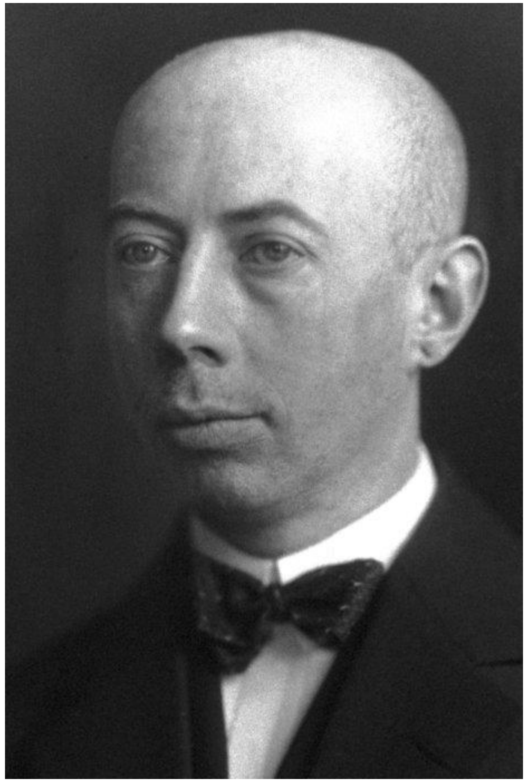
Albert Einstein

1921: “for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect”

Nobel Laureates: Quantum Mechanics

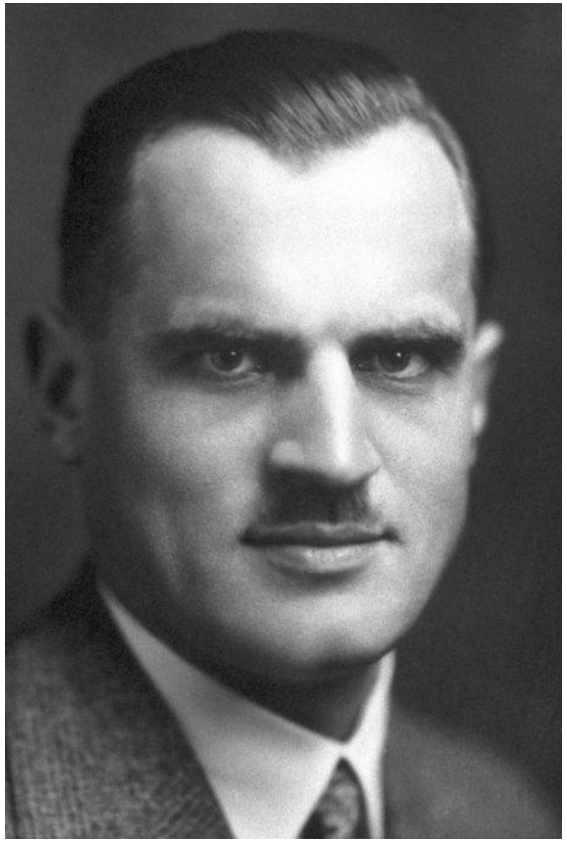
James Franck

1925: “for their discovery of the laws governing the impact of an electron upon an atom”



G. L. Hertz

Nobel Laureates: Quantum Mechanics



Arthur Holly Compton

1927: “for his discovery of the effect named after him”

Nobel Laureates: Quantum Mechanics

1929: “for his discovery of the wave nature of electrons”

Prince Louis-Victor Pierre Raymond de Broglie

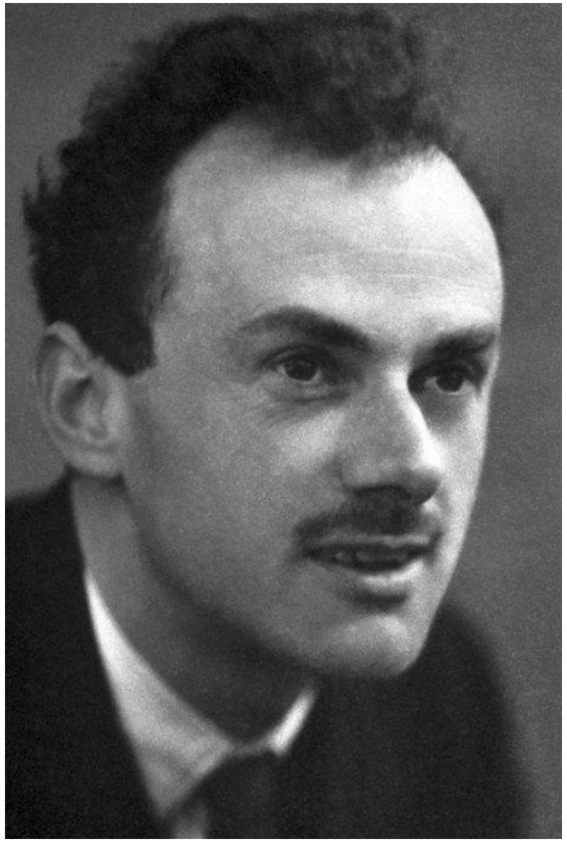
Nobel Laureates: Quantum Mechanics

Werner Karl Heisenberg

1932: “for the creation of quantum mechanics, the application of which has, inter alia, led to the discovery of the allotropic forms of hydrogen”

Nobel Laureates: Quantum Mechanics

1933: "for the discovery of new productive forms of atomic theory"



Erwin Schrödinger

Paul Adrien Maurice Dirac

Nobel Laureates: Quantum Mechanics

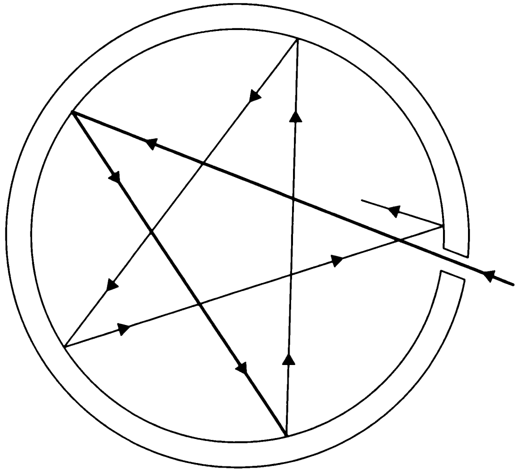
1937: “for their experimental discovery of the diffraction of electrons by crystals”

Clinton Joseph Davisson

George Paget Thomson

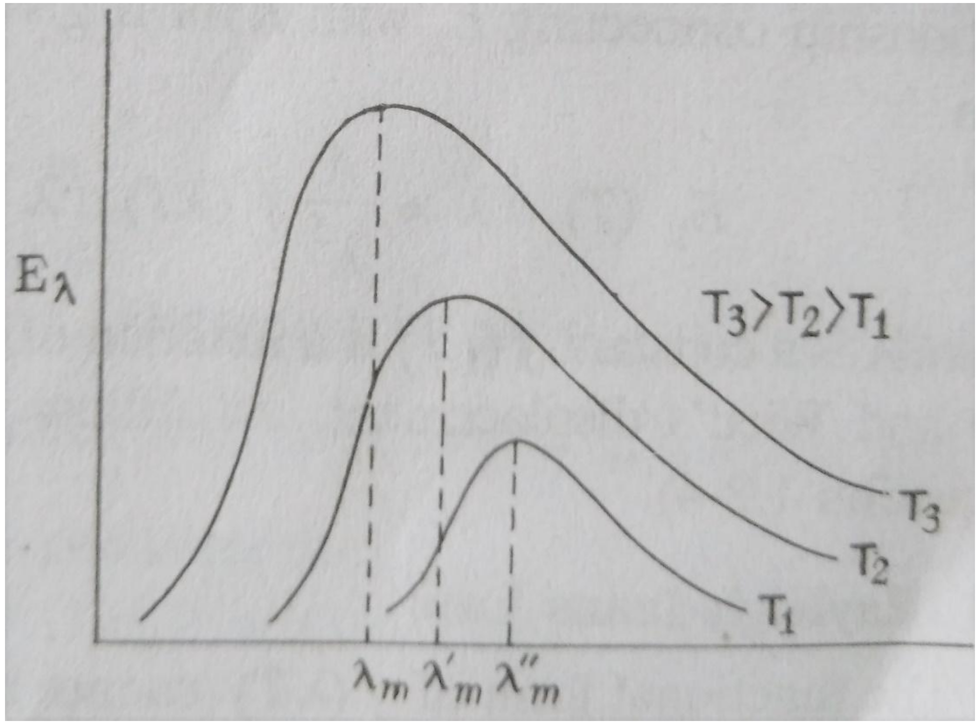
Blackbody Radiation

- Every body absorbs and emits radiation – intensity depends on wavelength and temperature
- **Kirchoff's Law:** At a given wavelength and at a given temperature, the ratio of the emissive power and absorptive power of a body is constant and is equal the emissive power of a perfect blackbody at that wavelength and at that temperature
- **(Perfect) Blackbody:** A body that absorbs the whole of incident radiation
- Emissive power of a perfect blackbody is independent of the nature of the body



Realization of perfect blackbody

Blackbody Radiation



- Blackbody emits radiation at all wavelengths. Intensity distribution depends on wavelength of the radiation and the temperature of the blackbody.
- The intensity of radiation at given temperature is maximum at a particular wavelength (λ_m). As the temperature is increased, λ_m shifts to shorter wavelengths–**Wien's displacement law:** $\lambda_m T$ =constant= 0.2896 cm-K.
- As the temperature of the blackbody increases, the intensity of radiation increases for all wavelengths. The total energy of radiation (E) increases with fourth power of the absolute temperature (T) of the blackbody – **Stefan's law:** $E \propto T^4$.

Blackbody Radiation

Wien's Law

- An empirical relationship proposed from thermodynamical considerations:

$$E_\lambda d\lambda = \frac{a}{\lambda^5} f(\lambda, T) d\lambda$$

- The functional form, though cannot be determined from thermodynamics, Wien's law includes Wien's displacement law and Stefan's law

- Wien proposed a functional form of $f(\lambda, T) \sim \exp(-b/\lambda T)$:

$$E_\lambda d\lambda = \frac{a}{\lambda^5} \exp(-b/\lambda T)\, d\lambda$$

Blackbody Radiation

Rayleigh-Jeans Law

- It is based on a classical model which assumes the radiation system composed of a collection of linear harmonic oscillator having their average energy kT (where k is the Boltzmann constant).
- The number oscillators per unit volume emitting radiation of wavelengths $\lambda \rightarrow \lambda + d\lambda$ is

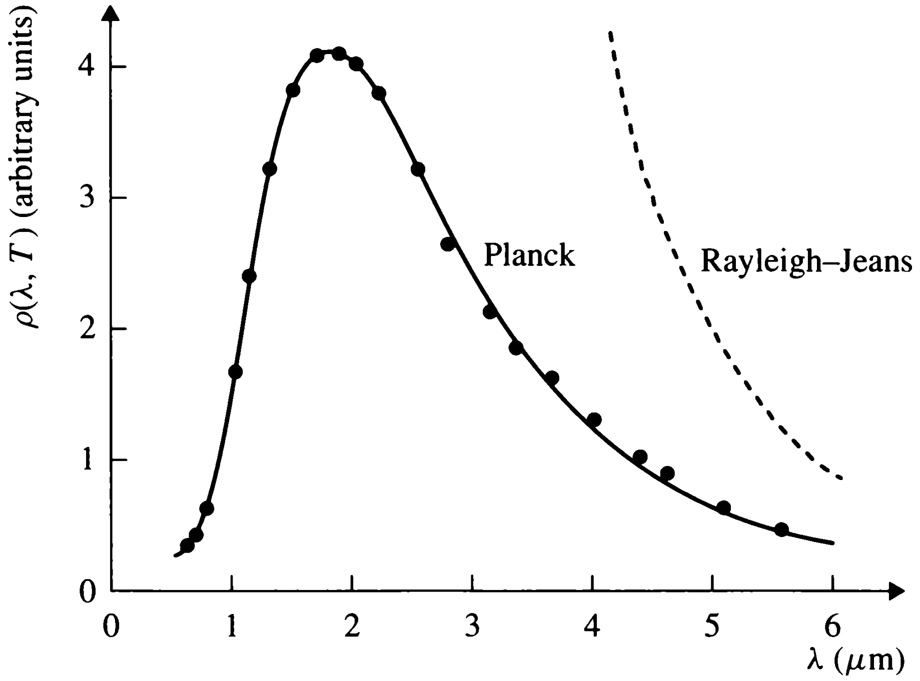
$$n_\lambda d\lambda = \frac{8\pi}{\lambda^4} d\lambda$$

- Hence Rayleigh-Jeans formula for the blackbody radiation characteristics is

$$E_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda$$

Blackbody Radiation

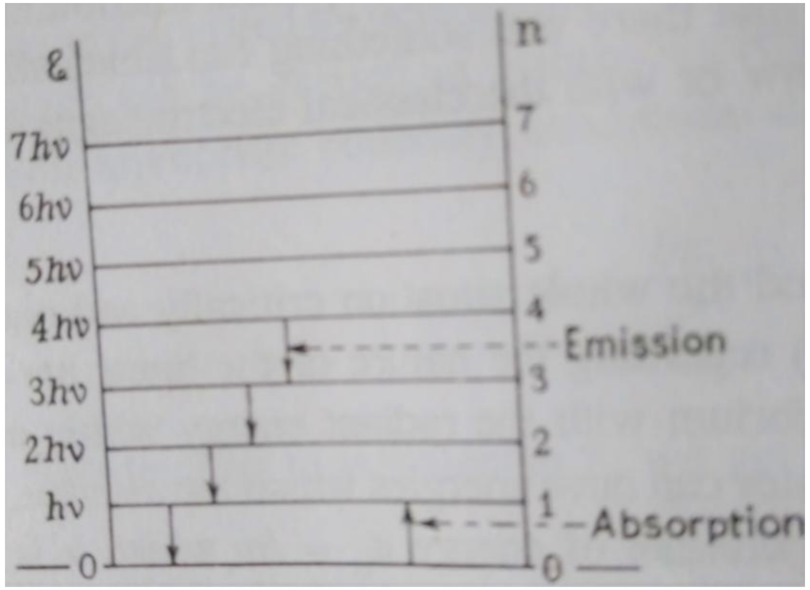
Limitation: Rayleigh-Jeans law is found to be consistent with observed blackbody radiation characteristics at higher wavelengths. However, a serious disagreement is observed at short wavelength: as $\lambda \to 0$, $E_\lambda \to 0$ – *ultraviolet catastrophe*.



Blackbody Radiation

Planck's Quantum Hypothesis

- Simple harmonic oscillator can have only discrete energies which are integral multiple of a finite quantum of energy $\varepsilon_0 = h\nu$ (h is the Planck's constant = 6.626×10^{-34} J-s, and ν is the frequency of the oscillator): $\varepsilon_n = n\varepsilon_0 = nh\nu, n = 0, 1, 2, \ldots$



- Oscillator emits/absorbs radiation in form of energy packet – photon. Each photon has energy $E = h\nu = hc/\lambda$. Photons carry momentum. The momentum of each photon is $p = E/c = h\nu/c$.

Blackbody Radiation

Planck's Law of Radiation

- The number of oscillators in an energy state $\varepsilon_n = nh\nu$ in equilibrium temperature T is determined by Maxwell-Boltzmann distribution:

$$N_n = N_0 \exp(-\varepsilon_n/kT) = N_0 \exp(-nh\nu/kT)$$

- Average energy of the oscillator:

$$\bar{\varepsilon} = \frac{\sum_{n=0}^{\infty} N_n \varepsilon_n}{\sum_{n=0}^{\infty} N_n} = \frac{\sum_{n=0}^{\infty} N_0 \varepsilon_n \exp(-\varepsilon_n/kT)}{\sum_{n=0}^{\infty} N_0 \exp(-\varepsilon_n/kT)} = \frac{\sum_{n=0}^{\infty} nh\nu \exp(-nh\nu/kT)}{\sum_{n=0}^{\infty} \exp(-nh\nu/kT)}$$

$$= \frac{h\nu x(1+2x+3x^2+4x^3+\ldots)}{(1+x+x^2+x^3+\ldots)}, \quad x = \exp(-h\nu/kT)$$

Blackbody Radiation

Planck's Law of Radiation

$$\bar{\varepsilon} = h\nu x \frac{(1-x)^{-2}}{(1-x)^{-1}} = \frac{h\nu x}{1-x} = \frac{h\nu}{(1/x)-1} = \frac{h\nu}{\exp(h\nu/kT)-1}$$

Number oscillators per unit volume emitting radiation of frequencies $\nu \to \nu + d\nu$: $n_\nu d\nu = \frac{8\pi}{c^3}\nu^2 d\nu$

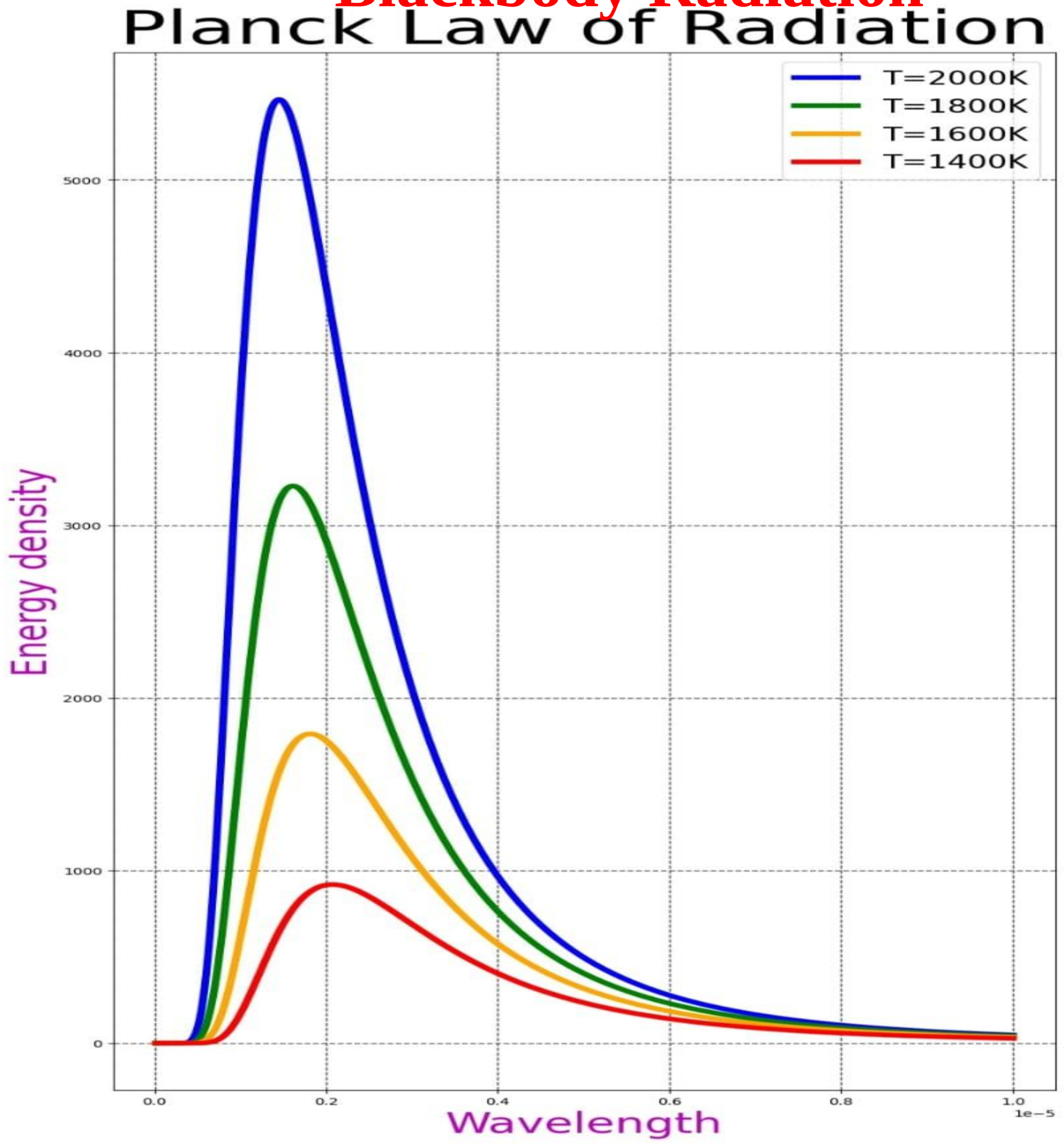
Energy density of radiation within frequencies between ν and $\nu + d\nu$:

$$u_\nu d\nu = (n_\nu d\nu)\bar{\varepsilon} = \left(\frac{8\pi}{c^3}\nu^2 d\nu\right)\bar{\varepsilon} = \frac{8\pi}{c^3}\frac{h\nu^3}{\exp(h\nu/kT)-1}d\nu$$

Energy density of radiation within wavelengths between λ and $\lambda + d\lambda$:

$$u_\lambda d\lambda = (n_\lambda d\lambda)\bar{\varepsilon} = \left(\frac{8\pi}{\lambda^4}d\lambda\right)\bar{\varepsilon} = \frac{8\pi hc}{\lambda^5}\frac{1}{\exp(hc/\lambda kT)-1}d\lambda$$

Blackbody Radiation



Blackbody Radiation

Planck's Law → Wien's Law

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{\exp(hc/\lambda kT)-1}d\lambda$$

In the short wavelength limit and lower temperature, λT is small i.e. $hc/\lambda kT \gg 1$ implying

$$\exp(hc/\lambda kT)-1 \approx \exp(hc/\lambda kT)$$

$$\Rightarrow u_\lambda d\lambda \approx \frac{8\pi hc}{\lambda^5}\exp(-hc/\lambda kT)d\lambda$$

$$= \frac{a}{\lambda^5}\exp(-b/\lambda T); \quad a = 8\pi hc, b = hc/k$$

Blackbody Radiation

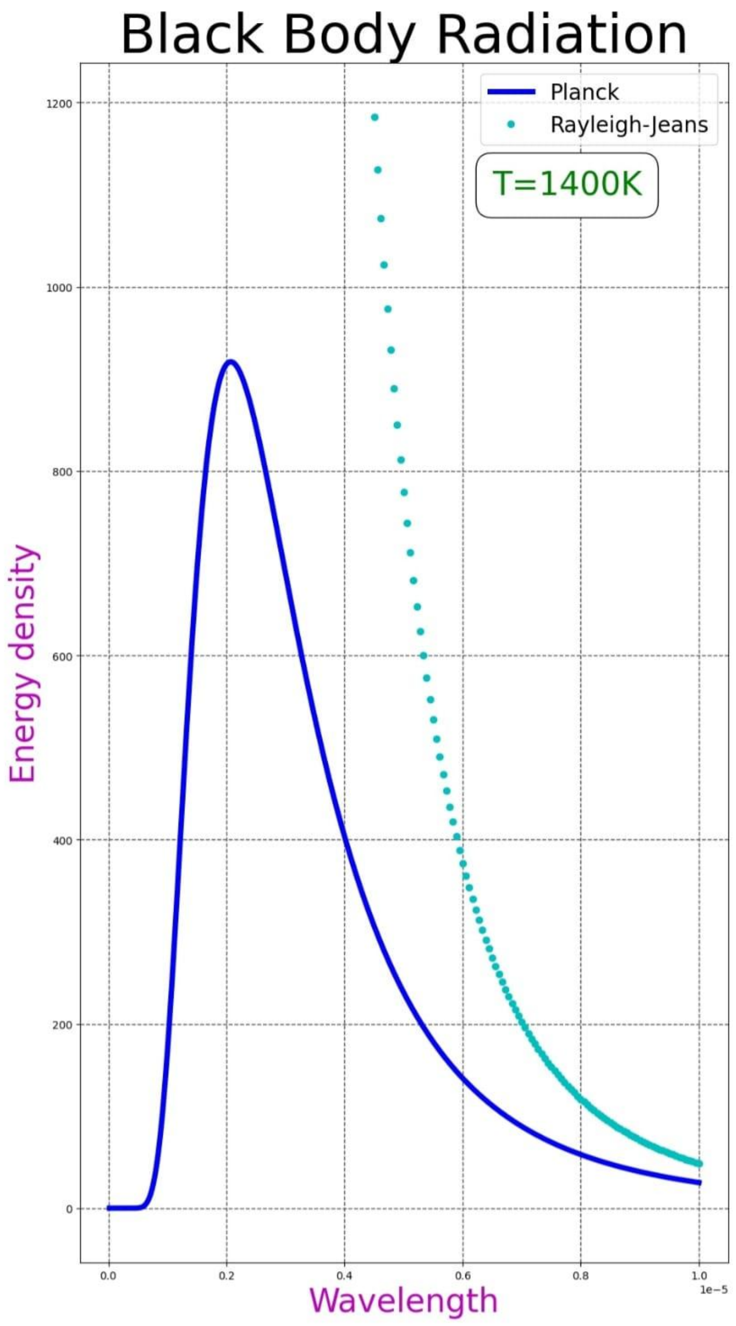
Planck's Law → Rayleigh-Jeans Law

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{\exp(hc/\lambda kT)-1}d\lambda$$

In the long wavelength limit and higher temperature, λT is large i.e., $hc/\lambda kT \ll 1$ implying

$$\exp(hc/\lambda kT) \approx 1 + \frac{hc}{\lambda kT}$$

$$\Rightarrow u_\lambda d\lambda \approx \frac{8\pi hc}{\lambda^5}\frac{\lambda kT}{hc}d\lambda = \frac{8\pi kT}{\lambda^4}d\lambda$$



Blackbody Radiation

Planck's Law → Wien's Displacement Law

Planck's radiation formula, i.e.,

$u_\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$, shows a maxima at the wavelength $\lambda = \lambda_m$ at which the denominator of the expression becomes minimum. The denominator can be written as, $z = \lambda^5\ (e^{hc/\lambda kT}-1)$

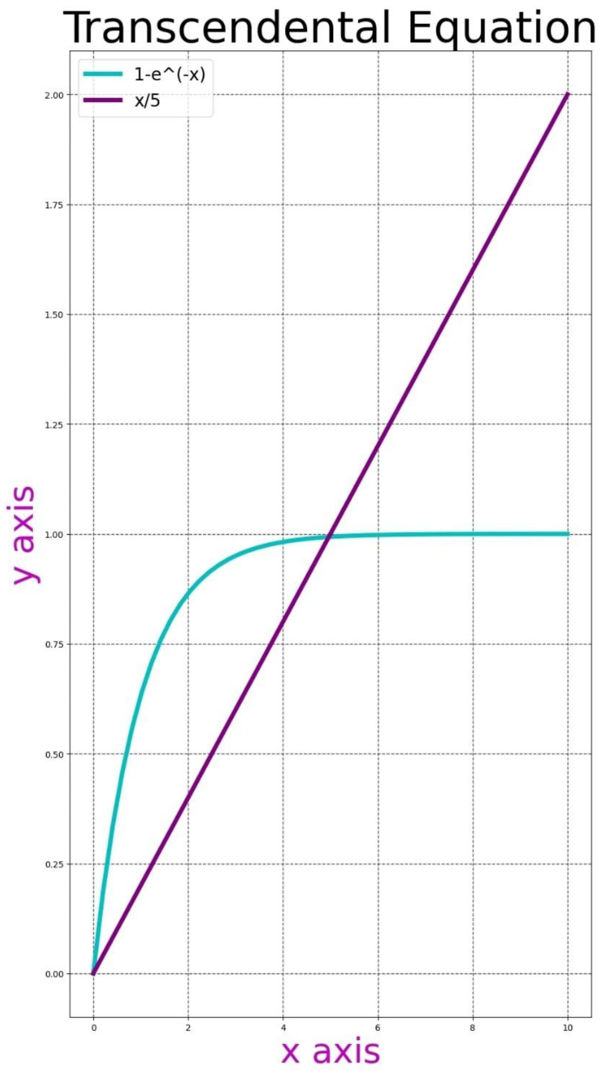
$$\therefore \frac{dz}{d\lambda} = 5\lambda^4\ (e^{hc/\lambda kT}-1) + \lambda^5 \cdot \frac{hc}{kT}\left\{\exp\left(\frac{hc}{\lambda kT}\right)\right\}\cdot\left(-\frac{1}{\lambda^2}\right) = 0, \text{ for } z \text{ to be minimum}$$

$$\Rightarrow 5\lambda^4\left\{\exp\left(\frac{hc}{\lambda kT}\right)-1\right\} = \lambda^3\,\frac{hc}{kT}\exp\left(\frac{hc}{\lambda kT}\right)$$

$$\text{or, } 1 - \exp\left(-\frac{hc}{\lambda kT}\right) = \frac{1}{5}\cdot\frac{hc}{\lambda kT}$$

$$\text{or, } 1 - \exp(-x) = \frac{x}{5}, \text{ where } x = \frac{hc}{\lambda kT} \qquad \ldots (1)$$

This is a transcendental equation which cannot be solved analytically. It can only be solved by graphical method.



If we put, $y = 1 - \exp(-x)$ and $y = \frac{x}{5}$ then the point of intersection of the two curves gives the solution at $x = 4.965$. We thus have,

$$\lambda_m T = \frac{hc}{kx} = \frac{hc}{4.965.k} = 0.29 \text{ cm-K} = \text{constant}$$

This is Wein's Displacement Law.

Blackbody Radiation

Problem 1: *The wavelength corresponding to maximum energy of solar radiation is 482 nm. Assuming the sun a black body, estimate the temperature of the sun.*

The wavelength corresponding to maximum energy of solar radiation is $\lambda_m = 482$ nm

If T is the temperature of the sun, according to Wien's displacement law

$$\lambda_m T = 0.2896 \text{ cm-K}$$

$$T = \frac{0.2896}{\lambda_m} \text{ K} = \frac{0.2896}{482 \times 10^{-7}} \text{ K} \approx 6008 \text{ K}$$

Blackbody Radiation

Planck's Law → Stefan's Law

The total energy density of the radiation emitted by a black body is given by,

$$u = \int_0^\infty u_\nu d\nu = \frac{8\pi h}{c^3}\int_0^\infty \nu^3 \frac{d\nu}{\exp\left(\frac{h\nu}{kT}\right)-1}$$

Let us consider $\frac{h\nu}{kT} = x$, thus, $d\nu = \left(\frac{kT}{h}\right) dx$

$$\therefore\ u = \frac{8\pi h}{c^3}\int_0^\infty \left(\frac{kT}{h}\right)^3 .x^3 \cdot \frac{1}{e^x-1}\left(\frac{kT}{h}\right)dx$$

$$= \frac{8\pi h}{c^3}\left(\frac{kT}{h}\right)^4 \int_0^\infty \frac{x^3}{e^x-1}dx$$

Here, $\int_0^\infty \frac{x^3}{e^x-1}dx = \frac{\pi^4}{15}$

$$\therefore\ u = \frac{8\pi h}{c^3}\left(\frac{kT}{h}\right)^4 \frac{\pi^4}{15} = aT^4$$

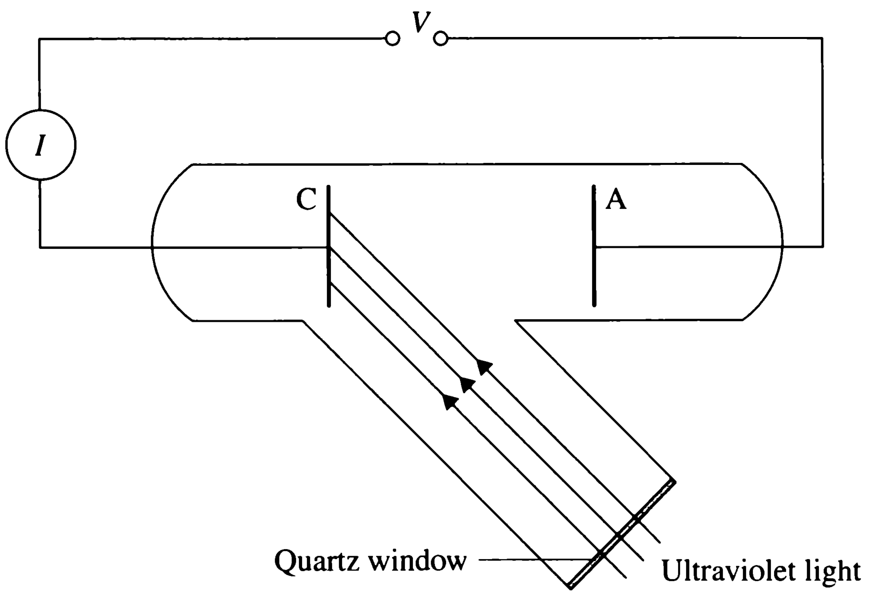
where, $a = \frac{8\pi^5 k^4}{15c^3h^3}$ = constant

The intensity E of the black body radiation is related to the energy density u as $E = \frac{c}{4}u = \frac{c}{4}.aT^4 = \sigma T^4$, σ = constant

Thus, we get the Stefan's Law.

Photoelectric Effect

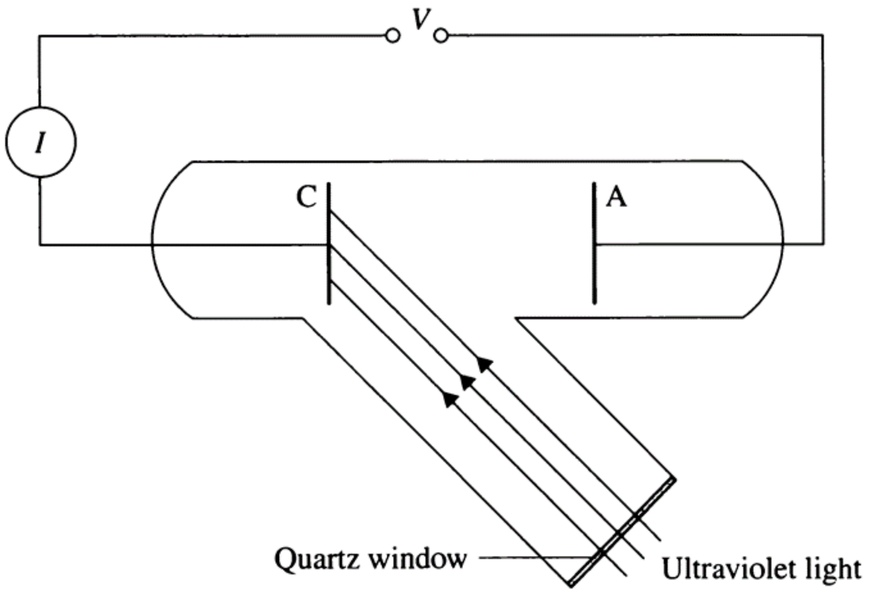
- Electron emission from metal surface (C) takes place when radiation of certain wavelength or below is incident on it
- Positively biased plate A collects the emitted photoelectrons and a photoelectric current (i) is produced
- As the positive bias of A is increased, the photoelectric current increases to reach a saturation value when all the photoelectrons emitted per second are collected by A



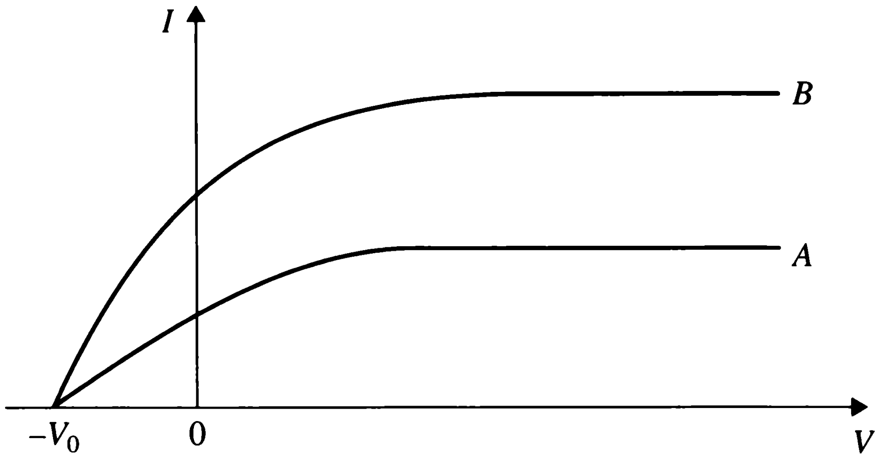
Quantum Mechanics, Bransden & Joachain

Photoelectric Effect

- As the electrons are emitted with some kinetic energy, some of them are able to reach A even when it is not biased or negatively biased. However, as the negative bias of A is increased, photoelectric current decreases. At some value $-V_0$, photoelectric current reduces to zero i.e., the electrons even emitted with maximum kinetic energy (for a given wavelength of the incident radiation), are unable to reach A overcoming the repulsive force. V_0 is called stopping potential.



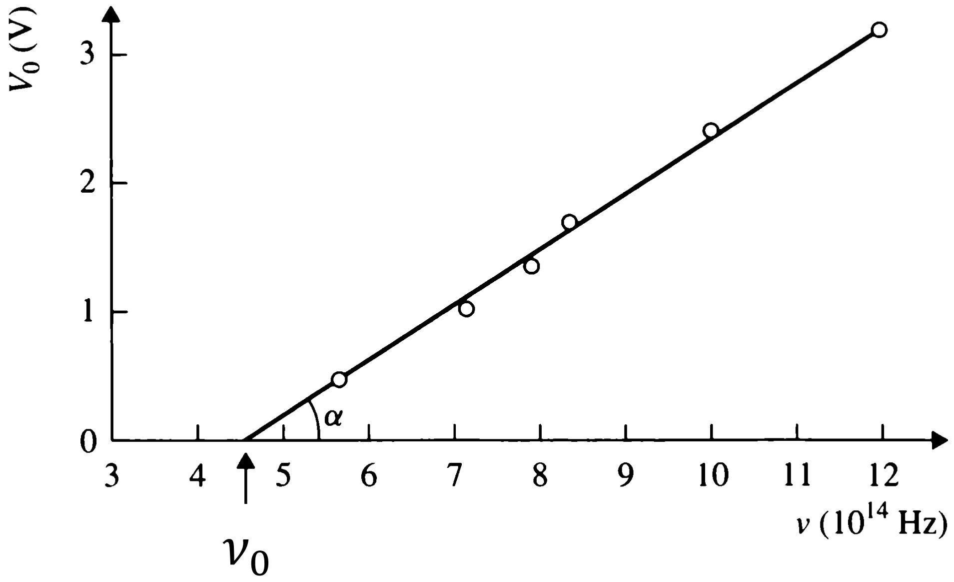
$$\frac{1}{2}mv_{max}^2 = eV_0$$

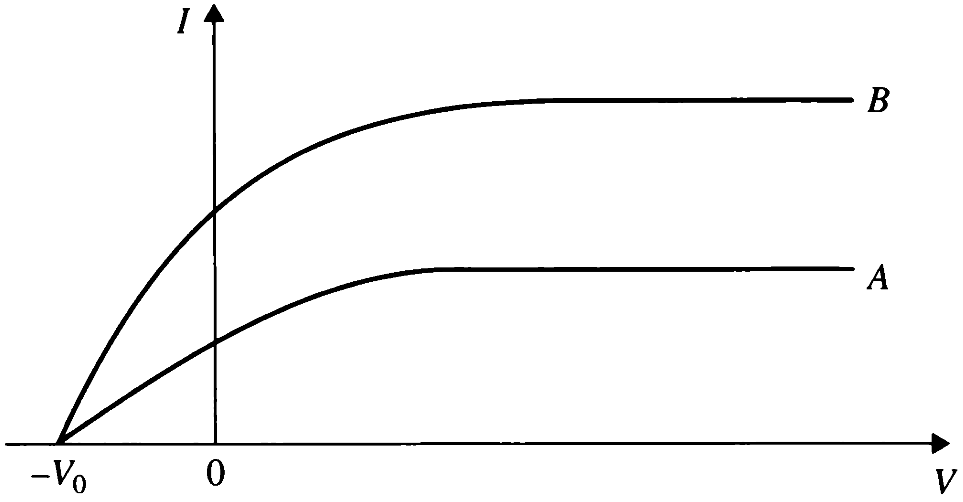


Photoelectric Effect

Characteristics

- Photoelectric effect does not take place below a certain frequency called the threshold frequency (ν_0) – which is the characteristic of the metal.
- The photoelectric current depends on the intensity of incident radiation. It increases with increase in the intensity of incident radiation.

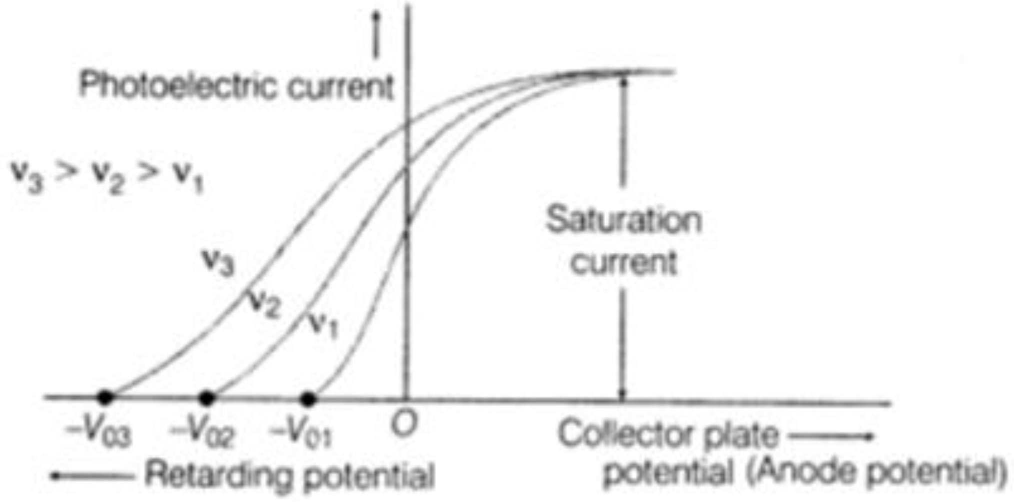


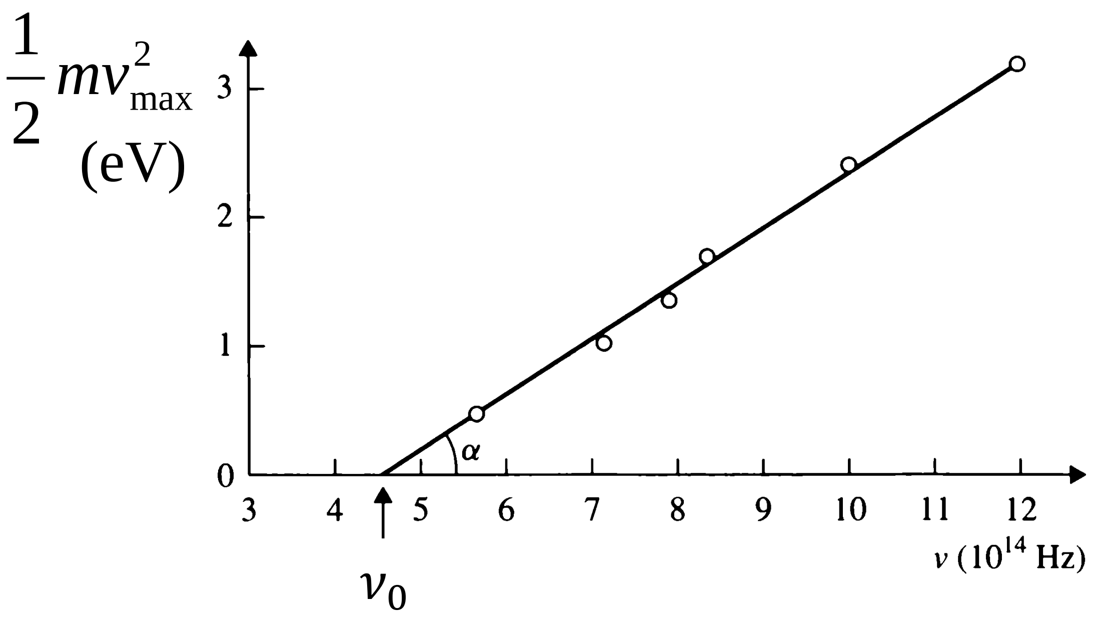


Photoelectric Effect

Characteristics

- The photoelectric current is independent of the wavelength of radiation.
- The maximum kinetic energy of the photoelectrons is independent of the intensity, but it increases with the frequency of the incident radiation.
- Photoelectric effect is an instantaneous process.



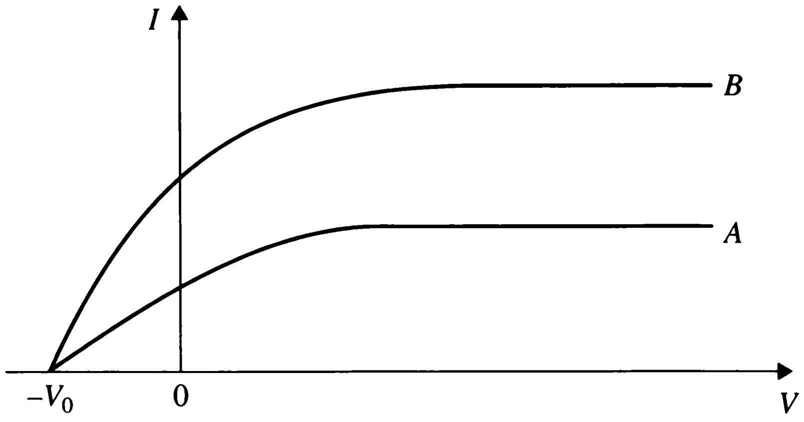


Quantum Mechanics, S. N. Ghoshal

Photoelectric Effect

Failure of Classical EM Theory

- Classical EMT: Light (Radiation) consists of mutually perpendicular oscillating electric and magnetic fields propagated as a transverse wave – intensity determined by the amplitude of electric and magnetic fields, not by the frequency or wavelength
- As light falls on electron, it is acted upon by the oscillating electric field and gains energy – larger the amplitude of electric field (i.e., larger the intensity of light), larger is the energy transfer. Thus, the energy of the emitted photoelectron (measured in terms of the stopping potential) should depend on the intensity of the incident light, contrary to the actual observation



Quantum Mechanics, S. N. Ghoshal

Photoelectric Effect

Failure of Classical EM Theory

- According to EMT, the velocity of the emitted photoelectrons should be independent of the frequency of the incident light. As light passes by an electron, it collects energy from each passing wave and when it collects enough energy after sufficiently long time, it should be emitted from the metal surface. Thus, classical EMT rules out the concept of threshold frequency and the photoelectric phenomenon as instantaneous process
- Incident EM wave acts equally on electrons of the metal surface. Given sufficient time, all electrons should be able to collect enough energy to be emitted – thus photoelectric current should be independent of the intensity of the incident radiation contrary to our actual observation

Photoelectric Effect

Einstein's Photoelectric Equation

- Einstein's Light Quantum Hypothesis: The light is emitted from a source in the form of bundles of energy quantum – $h\nu$, known as light quantum or photon
- The intensity of light is thus the number of photons emitted per second
- When an electron absorbs a photon it gains the energy of photon $h\nu$ and if $h\nu > \phi_0$, the work function of the metal (height of the potential barrier at the metal surface, the minimum energy required for an electron to emit from the surface), the surplus energy $(h\nu - \phi_0)$ is carried out by the electron as kinetic energy. Thus

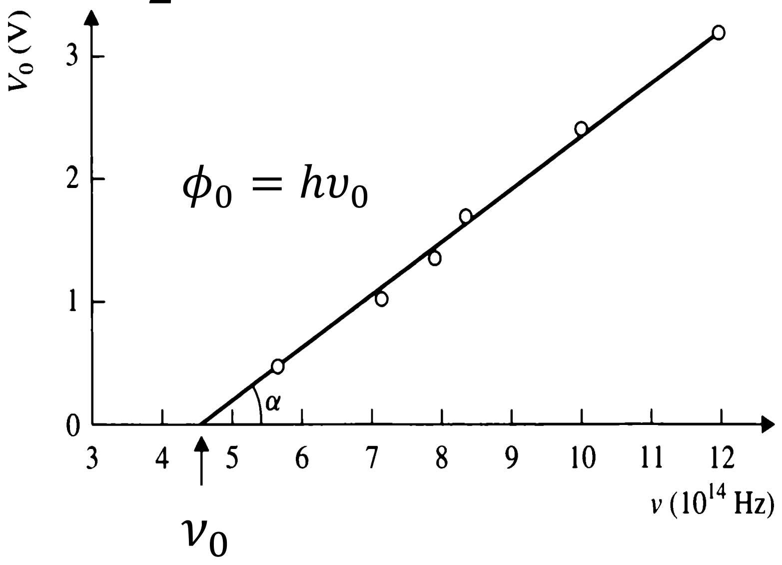
$$\frac{1}{2}mv_{max}^2 = eV_0 = h\nu - \phi_0$$

Photoelectric Effect

Einstein’s Photoelectric Equation

- Variation of stopping potential (maximum kinetic energy of photoelectrons) with frequency of incident radiation explained
- Threshold frequency explained: $\phi_0 = h\upsilon_0$
- As intensity of the incident radiation is the number of photons hitting per second, number of photoelectrons emitted per second i.e., photoelectric current is found to be proportional to the intensity
- Emission of photoelectron is an instantaneous process as the electron emits instantaneously as it absorbs incident photon

$$\frac{1}{2}mv_{max}^2 = eV_0 = h\nu - h\upsilon_0$$



Photoelectric Effect

Problem 2: *Find the number of photons emitted per second by a 40 Watt source of light of wavelength 6000 Å.* [CU – 2015]

Let us consider N number of photons each of frequency ν and wavelength λ are emitted per second from the source. Since each photon carries an energy $h\nu$, the energy emitted from the source per second i.e., power

$$P = Nh\nu = \frac{Nhc}{\lambda}$$

or, $$N = \frac{P\lambda}{hc} = \frac{40 \times 6000 \times 10^{-10}}{6.626 \times 10^{-34} \times 3 \times 10^{8}} \text{ per sec} = 1.21 \times 10^{20} \text{ per sec}$$

Photoelectric Effect

Problem 3: *Monochromatic radiation at wavelength 300 nm is incident on a piece of barium (photoelectric work function 2.5 eV). Will there be any photoelectric emission? If yes, find the maximum kinetic energy of the photoelectron. Also find maximum velocity of the photoelectrons. What is the stopping potential in this case?*

The energy of the incident photon $E = h\nu = \frac{hc}{\lambda} = \frac{4.136\times10^{-15}\times3\times10^{8}}{300\times10^{-9}}$ eV $= 4.136$ eV

Since $E > \phi_0$, there will be photoelectric emission.

The maximum kinetic energy of the photoelectron

$$E_{\text{k max}} = E - \phi_0 = (4.136 - 2.5\,)\text{ eV} = 1.636\text{ eV}$$

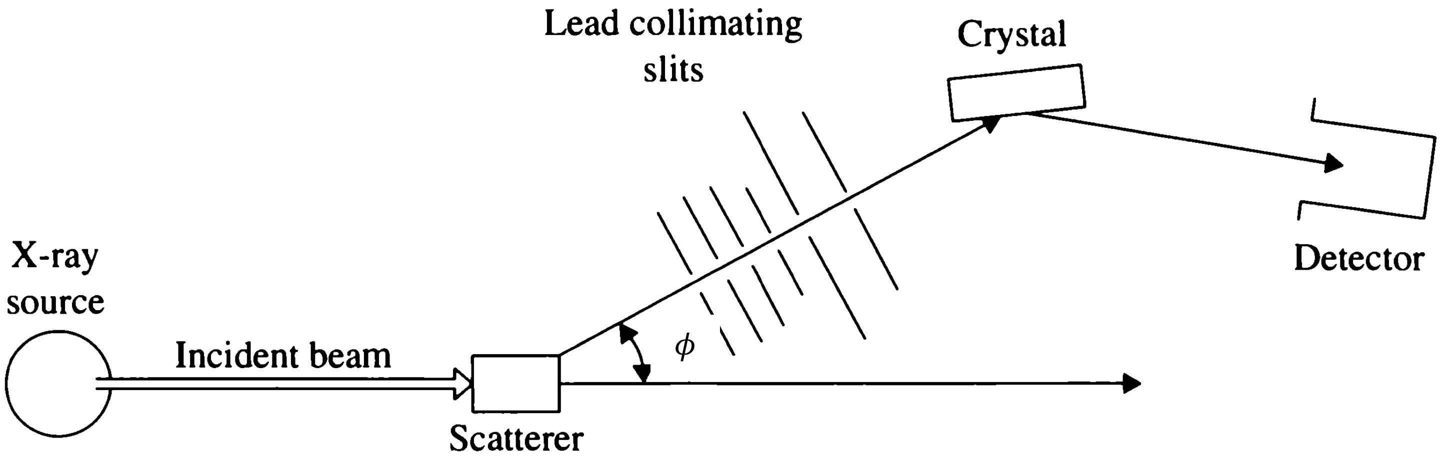
If v_{max} is the maximum velocity of the photoelectrons, $E_{\text{k max}} = \frac{1}{2}mv_{\text{max}}^2$

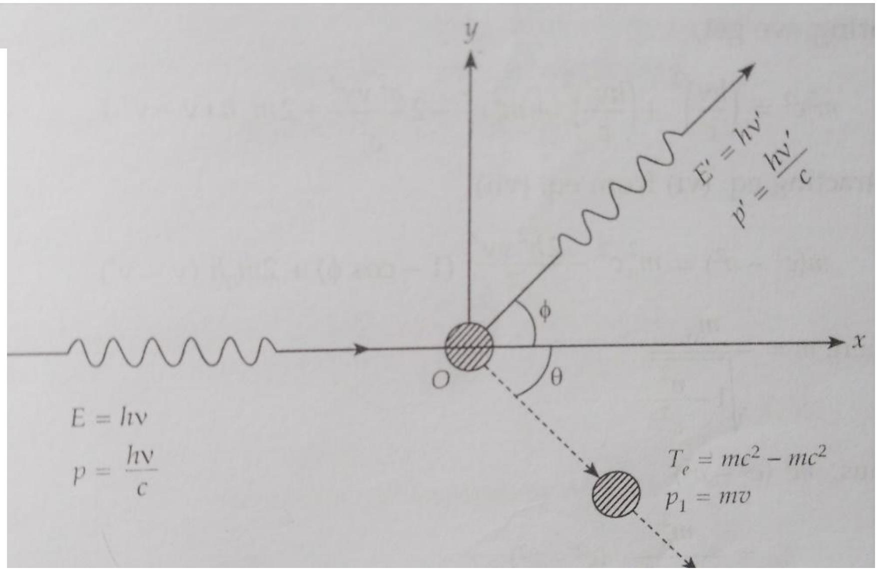
$$\text{or, } v_{\text{max}} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\times1.636\times1.6\times10^{-19}}{9.1\times10^{-31}}}\text{ m/s} = 7.57\times10^{5}\text{ m/s}$$

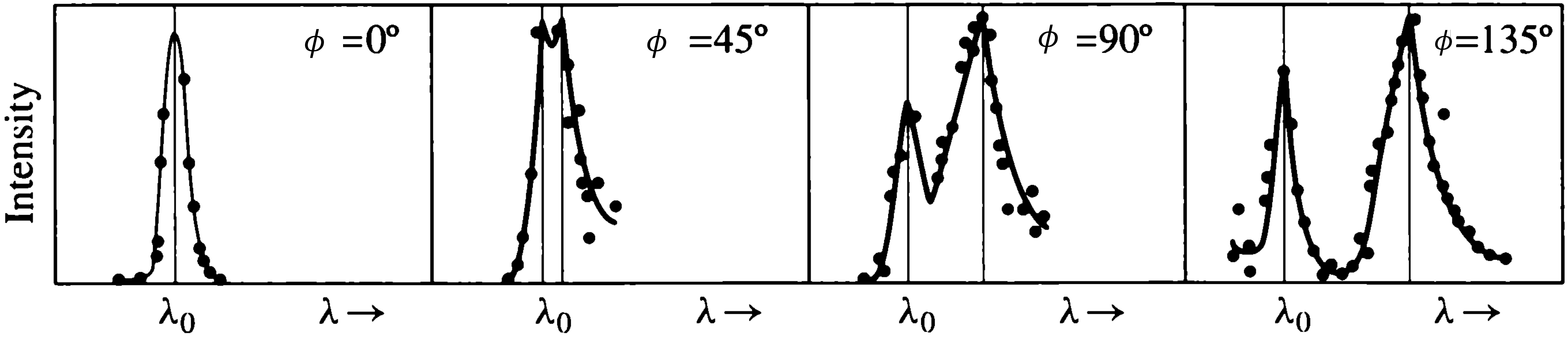
If V_0 is the stopping potential, $eV_0 = E_{\text{k max}} = 1.636$ eV, i.e. $V_0 = 1.636$ volt.

Compton Effect

Compton effect is a phenomenon of scattering of high energy electromagnetic radiation like X −ray and γ −ray with free or loosely bound electrons







Compton Effect

Compton Shift:

Now, applying the law of conservation of energy,

$$h\nu = h\nu' + T_e = h\nu' + (mc^2 - m_0c^2) \quad \text{... (i)}$$

Applying the law of conservation of linear momentum,

(along X) : $$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\phi + mv\cos\theta \quad \text{... (ii)}$$

(along Y) : $$0 = \frac{h\nu'}{c}\sin\phi - mv\sin\theta \quad \text{... (iii)}$$

Eq. (ii) $\Rightarrow$ $$mv\cos\theta = \frac{h\nu}{c} - \frac{h\nu'}{c}\cos\phi \quad \text{... (iv)}$$

Eq. (iii) $\Rightarrow$ $$mv\sin\theta = \frac{h\nu'}{c}\sin\phi \quad \text{... (v)}$$

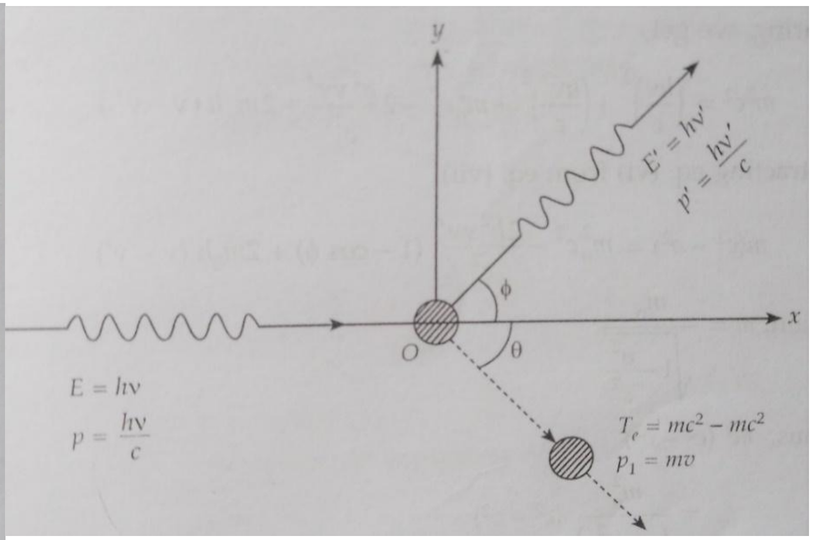
Squaring and adding equations (iv) and (v),

$$m^2v^2 = \left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 - 2\frac{h^2\nu\nu'}{c^2}\cos\phi \quad \text{... (vi)}$$

From equation (i) $\Rightarrow$

$$mc^2 = h\nu - h\nu' + m_0c^2$$

$$\Rightarrow \quad mc = \frac{h\nu}{c} - \frac{h\nu'}{c} + m_0c$$



Problems in Modern Physics, P. Mandal

Compton Effect

Compton Shift:

Squaring, we get,

$$m^2c^2 = \left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 + m_0^2c^2 - 2\frac{h^2\nu\nu'}{c^2} + 2m_0h(\nu - \nu') \quad \text{... (vii)}$$

Subtracting eq. (vi) from eq. (vii)

$$m(c^2 - v^2) = m_0^2c^2 - \frac{2h^2\nu\nu'}{c^2}(1 - \cos\phi) + 2m_0h(\nu - \nu')$$

Again, $m = \dfrac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

Thus, $m^2(c^2 - v^2)$

$$= \frac{m_0^2}{\left(1 - \dfrac{v^2}{c^2}\right)}(c^2 - v^2)$$

$$= \frac{m_0^2c^2}{(c^2 - v^2)}(c^2 - v^2)$$

$$= m_0^2c^2$$

$$\therefore\ m_0^2c^2 = m_0^2c^2 - \frac{2h^2\nu\nu'}{c^2}(1 - \cos\phi) + 2m_0h(\nu - \nu')$$

$$\Rightarrow 2m_0h(\nu - \nu') = \frac{2h^2\nu\nu'}{c^2}(1 - \cos\phi)$$

$$\Rightarrow \frac{\nu - \nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1 - \cos\phi)$$

$$\Rightarrow c\left(\frac{1}{\nu'} - \frac{1}{\nu}\right) = \frac{h}{m_0c}(1 - \cos\phi)$$

$$\Rightarrow \lambda' - \lambda = \lambda_c(1 - \cos\phi)$$

where, $\lambda_c = \dfrac{h}{m_0c}$ is known as Compton wavelength

$$\Rightarrow \Delta\lambda = 2\lambda_c\sin^2\frac{\phi}{2}$$

$\Delta\lambda$ is the shift in wavelength.

Compton Effect

Electron Scattering Direction:

$$\tan\theta = \frac{\frac{h\nu'}{c}\sin\phi}{\frac{h\nu}{c} - \frac{h\nu'}{c}\cos\phi}$$

$$= \frac{\nu'\sin\phi}{\nu - \nu'\cos\phi}$$

$$= \frac{\sin\phi}{\frac{\nu}{\nu'} - \cos\phi} \quad \text{... (i)}$$

Again, from equation (viii) in Solved Problem 14

$$\frac{1}{\nu'} = \frac{1}{\nu} + \frac{h}{m_0c^2}(1-\cos\phi)$$

$$\therefore \quad \frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1-\cos\phi)$$

$$\Rightarrow \quad \frac{1}{\nu}\left(\frac{\nu}{\nu'} - 1\right) = \frac{h}{m_0c^2}(1-\cos\phi)$$

$$\Rightarrow \quad \frac{\nu}{\nu'} - 1 = \frac{h\nu}{m_0c^2}(1-\cos\phi)$$

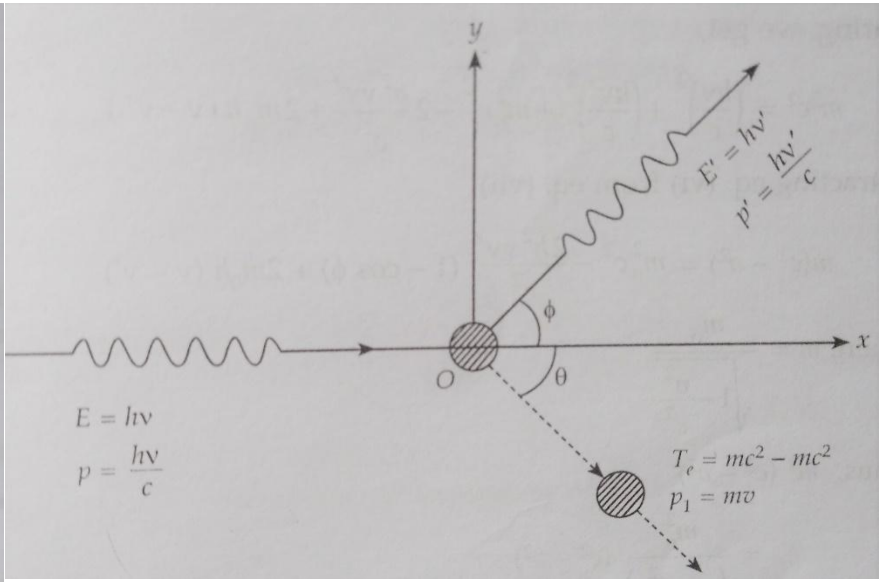
$$\Rightarrow \quad \frac{\nu}{\nu'} = 1 + \alpha(1-\cos\phi) \text{ where, } \alpha = \frac{h\nu}{m_0c^2} \text{ and } \nu = \text{frequency}$$

Substituting this value of $\frac{\nu}{\nu'}$ in eq. (i)

$$\tan\theta = \frac{\sin\phi}{1+\alpha(1-\cos\phi)-\cos\phi}$$

$$= \frac{2\sin\frac{\phi}{2}\cos\frac{\phi}{2}}{(1-\cos\phi)(1+\alpha)} = \frac{2\sin\frac{\phi}{2}\cos\frac{\phi}{2}}{2\sin^2\frac{\phi}{2}(1+\alpha)}$$

$$= \frac{\cot\frac{\phi}{2}}{1+\frac{h\nu}{m_0c^2}} \quad \left[\because \alpha = \frac{h\nu}{m_0c^2}\right]$$



while the photon is scattered at angles between 0° to 180°, the recoil electron is emitted at angles between 0° to 90°.

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Compton Effect

Energy of Recoiled Electron:

$$T_e = h\nu - h\nu' \quad \text{... (i)}$$

Again, the Compton shift,

$$\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\phi)$$

$$\Rightarrow \frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0 c^2}(1 - \cos\phi)$$

$$\Rightarrow \frac{\nu}{\nu'} - 1 = \frac{h\nu}{m_0 c^2}(1 - \cos\phi)$$

$$\Rightarrow \frac{\nu}{\nu'} = 1 + \frac{h\nu}{m_0 c^2}(1 - \cos\phi)$$

$$\Rightarrow \nu' = \frac{\nu}{1 + \alpha(1 - \cos\phi)} \quad \text{where, } \alpha = \frac{h\nu}{m_0 c^2}$$

From equation (i),

$$T_e = h\nu - h\nu'$$

$$= h\nu\left[\frac{1 + \alpha(1 - \cos\phi) - 1}{1 + \alpha(1 - \cos\phi)}\right]$$

$$= \frac{h\nu \cdot \frac{h\nu}{m_0 c^2} \cdot 2\sin^2\frac{\phi}{2}}{1 + \frac{h\nu}{m_0 c^2} \cdot 2\sin^2\frac{\phi}{2}}$$

$$\therefore T_e = \frac{2 \cdot \frac{h^2\nu^2}{m_0 c^2}\sin^2\frac{\phi}{2}}{1 + \frac{2h\nu}{m_0 c^2}\sin^2\frac{\phi}{2}}$$

Clearly, T_e is maximum at $\phi = 180°$

$$\therefore T_{e,\,max} = \frac{\frac{2h^2\nu^2}{m_0 c^2}}{1 + \frac{2h\nu}{m_0 c^2}}$$

$$= \frac{2h^2\nu^2}{2h\nu + m_0 c^2}$$

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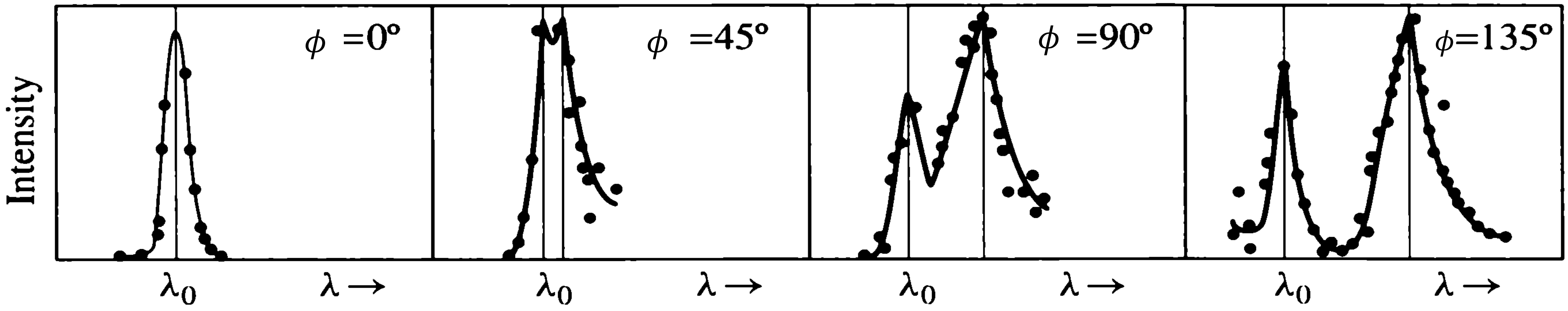
Compton Effect

Why two peaks?



- Peak at longer wavelength is due to Compton scattering from the electron which may be considered free (for a loosely bound electron, the binding energy is small as compared to the energy of the short wavelength X-ray or γ-ray photons)
- The other peak at the wavelength of the incident radiation (λ_0) is due to scattering from a tightly bound electron (core electron) as the recoil momentum is taken up by the entire atom

What is light?

“All the fifty years of conscious brooding have brought me no closer to the answer to the question, 'What are light quanta?' Of course today every rascal thinks he knows the answer, but he is deluding himself” – Albert Einstein (1951)

“Newton thought that light was made up of particles, but then it was discovered that it behaves like a wave. Later, however (in the beginning of the twentieth century), it was found that light did indeed sometimes behave like a particle. Historically, the electron, for example, was thought to behave like a particle, and then it was found that in many respects it behaved like a wave. So it really behaves like neither. Now we have given up. We say: 'It is like neither'. There is one lucky break, however-electrons behave just like light. The quantum behaviour of atomic objects (electrons, protons, neutrons, photons, and so on) is the same for all, they are all 'particle-waves', or whatever you want to call them” – Feynman