**ST. PAUL'S CATHEDRAL MISSION COLLEGE**
***DEPARTMENT OF MATHEMATICS***
***Class Test***
***Subject-CC-14***
(Topic: Numerical Differentiation and Integration )
*Six Semester*

**Full Marks: 20** **Time: 45 minutes**

Each question caries five marks

**Answer any four questions** **Date: 03.05.2025**



1. (a) What do you mean by the **degree of precision** of a mechanical quadrature formula?
   (b) For which quadrature rule is the degree of precision maximum?
   (c) Give a **geometrical interpretation** of the composite Simpson's one-third rule.
   (d) State the **error term** of Simpson's one-third rule and comment upon the degree of precision. (1+1+2+1 marks)

2. (a) The quadrature formula
$$\int_0^1 f(x)\,dx = \frac{1}{2}\left(f(x_0) + c_1 f(x_1)\right)$$
has degree of precision as high as possible. Determine $x_0$, $c_1$, and $x_1$.
   (b) Correct or justify: *Simpson's $\frac{3}{8}$ rule is slightly more accurate than Simpson's $\frac{1}{3}$ rule.* (3+2 marks)

3. (a) Establish the midpoint rule:
$$\int_{x_0}^{x_1} f(x)\,dx = hf\left(\frac{x_0+x_1}{2}\right) + \frac{h^3}{24}f''(\xi), \quad \text{where} \quad \xi \in (x_0, x_1), \quad h = x_1 - x_0.$$
   (b) Correct or justify: *For each integer $n > 3$, there exists a polynomial $P_n(x)$ of degree $n$ such that Simpson's $\frac{1}{3}$ rule integrates $P_n(x)$ exactly.*
   (Consider the interval of integration to be $[0, 1]$ for simplicity.) (3+2 marks)

4. Given a table of the function $f(x)$ at the points $x_r = x_0 + rh$ (where $h > 0$ and $r = 0, 1, 2, \ldots, n$), obtain a numerical differentiation formula for computing $f'(x_0)$, assuming an appropriate interpolation formula.

   Discuss: *"Large errors are often found in approximating derivatives from the polynomial interpolation formulae"*- discuss the main reasons behind it. (3 + 2 marks)

5. Calculate the approximate values of
$$\left(\frac{dy}{dx}\right)_{x=2.8} \quad \text{and} \quad \left(\frac{d^2y}{dx^2}\right)_{x=3.0}$$
using the following table:

| $x$ | $y$ |
|---|---|
| 2.0 | 0.6020 |
| 2.2 | 0.6848 |
| 2.4 | 0.7604 |
| 2.6 | 0.8299 |
| 2.8 | 0.8943 |
| 3.0 | 0.9542 |

(3 + 2 marks)

6. Explain the principle of numerical differentiation. Deduce Lagrange's numerical differentiation formula and deduce it in particular for computing the derivative at an interpolating point. (1+2+2 marks)

7. Deduce the Newton-Cotes formula to approximate

$$\int_a^b f(x)\,dx$$

by selecting the nodes $x_0 = a$, $x_1 = a+h$, and $x_2 = a+2h = b$. State, with justification, whether it is a closed type formula. (4+1 marks)