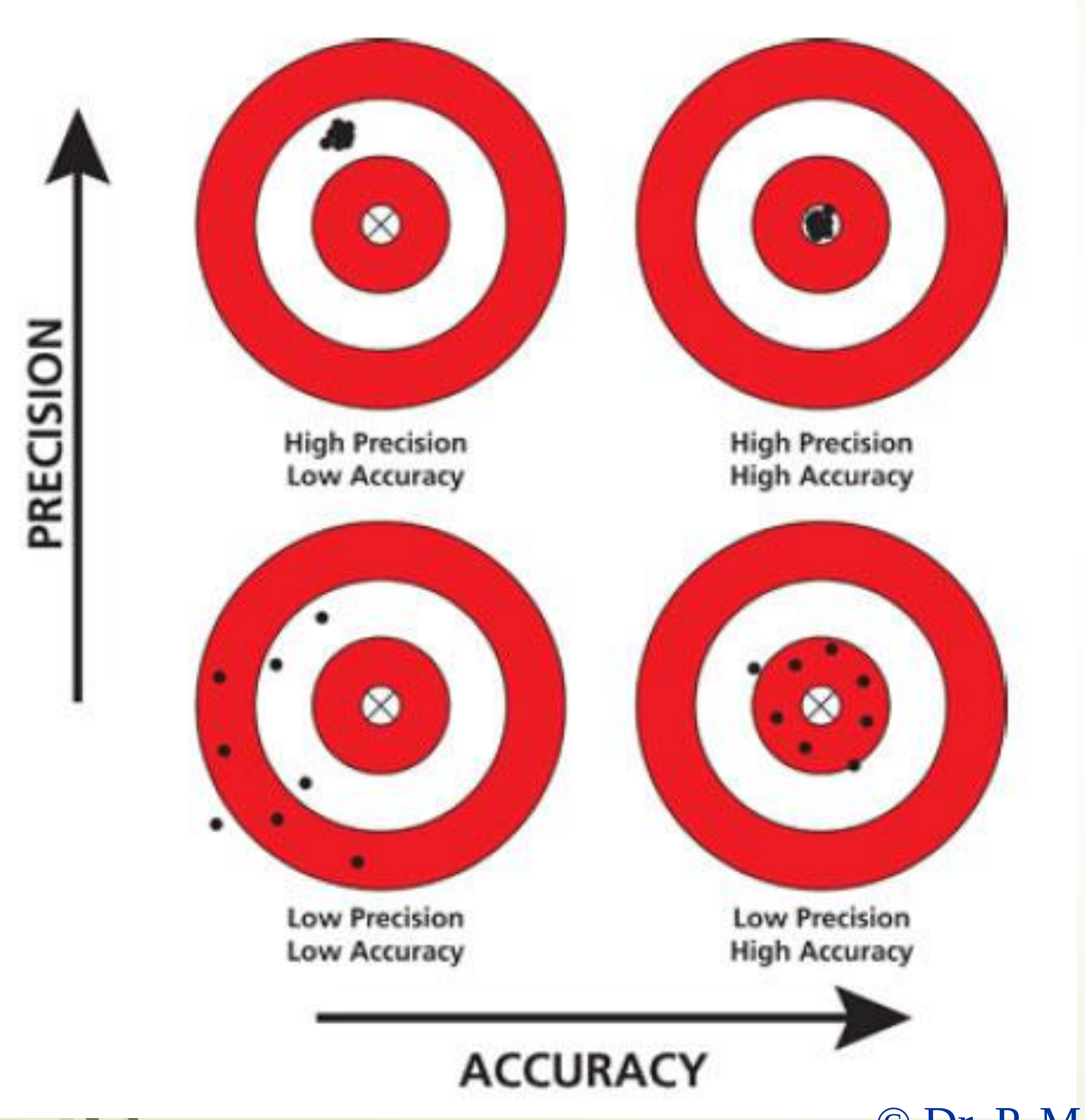
Fundamentals of Measurements

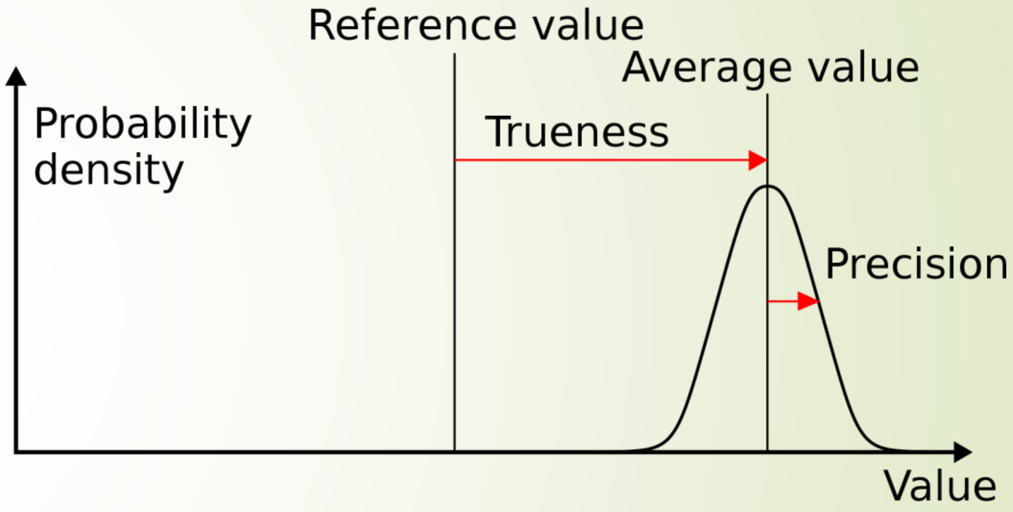
Why measurements?

- “*The heart of science is measurement*” - Erik Brynjolfsson
- “*An experiment is a question which science poses to Nature, and a measurement is the recording of Nature’s answer*” – Max Planck
- “*If you cannot measure it, you cannot improve it*” – Lord Kelvin
- “*There are two possible outcomes: If the result confirms the hypothesis, then you've made a measurement. If the result is contrary to the hypothesis, then you've made a discovery*” – E. Fermi
- “*I belong to those theoreticians who know by direct observation what it means to make a measurement. Methinks it were better if there were more of them*” – Erwin Schrödinger

Fundamentals of Measurements

Accuracy vs. Precision





Accuracy vs. Precision

Accuracy

- How close the measured value is to the 'true value' (best known value).
- Refers to the degree of conformity.
- Affected by systematic errors.

Precision

- How many times repeated measurements under unchanged conditions produce the same result.
- Refers to the degree of reproducibility or repeatability.
- Affected by random errors.

Mean, Median and Mode

Mean/Average: Arithmetic mean of a dataset (different from the geometric mean) is the sum of all values divided by the total number of values.

$$\langle x \rangle \equiv \bar{x} = x = \frac{x_1 + x_2 + x_3 + \cdots x_n}{n} = \frac{1}{n}\sum_{i}^{n} x_i \equiv \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

Examples: Calculate the arithmetic mean of 5, 6, 7, 8, 9, 10, 11

$$\langle x \rangle = \frac{5 + 6 + 7 + 8 + 9 + 10 + 11}{7} = 8$$

Problem: Calculate the arithmetic mean of the following data set:

Data (x)	5	10	15	20	25
Frequency (f)	5	2	2	3	4

Fundamentals of Measurements

Mean, Median and Mode

Median: Median is the middle value in a data set that has been ordered (in ascending or descending orders).

Examples: Time period of a pendulum (in second): 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4

Median: 2.1 s

Median of an odd-numbered ($n =$ odd) dataset: data that lies at the position $(n + 1)/2$.

Median of an even-numbered ($n =$ even) dataset: find two data, one at the position $n/2$ and the other at the position $1 + n/2$ and find their mean.

Example: Diameter of a wire measured with a screw gauge (in mm): 2.01, 2.02, 2.04, 2.06, 2.07, 2.08

Median: 2.05 mm

Mean, Median and Mode

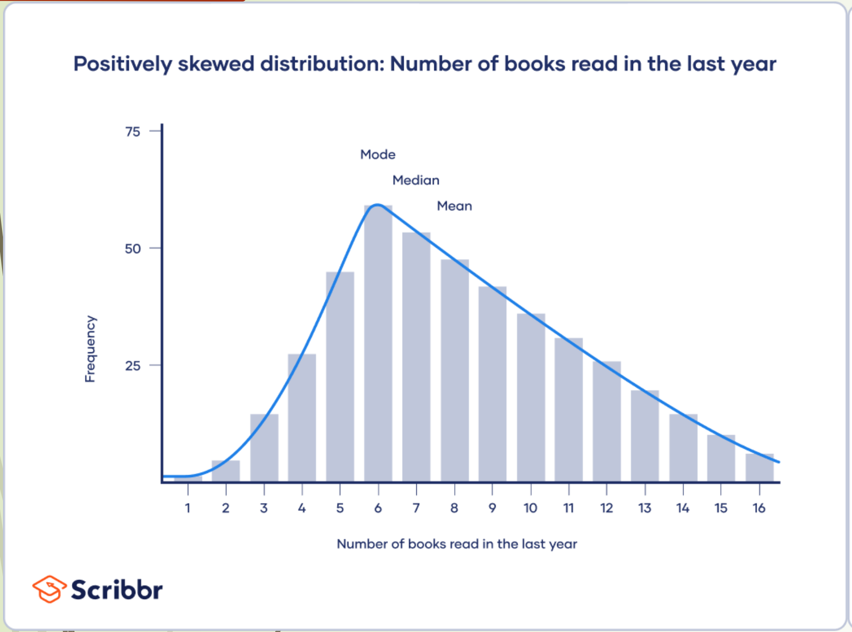
Mode: Mode in a dataset is/are the data with highest frequency of occurrence. In other words, mode is the most probable value.

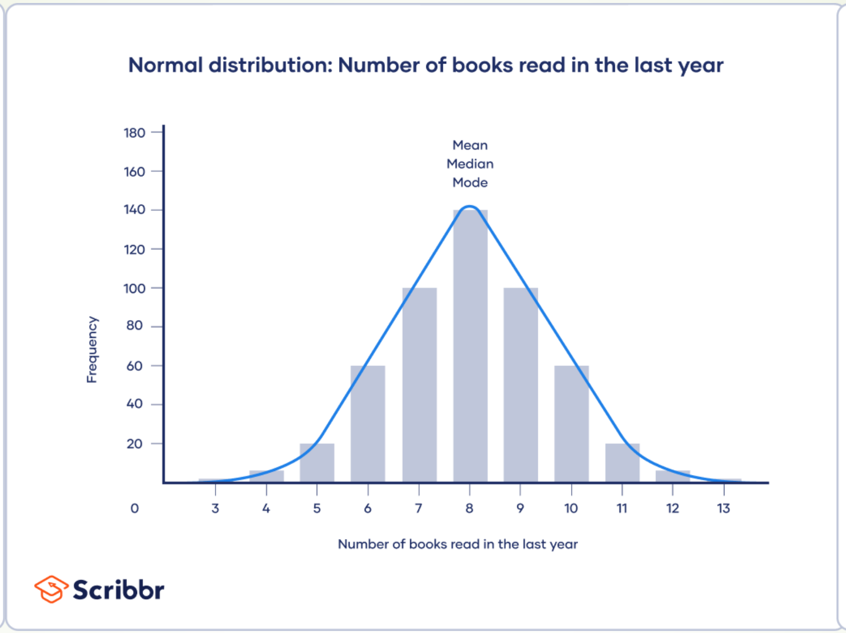
It's possible to have no mode, one mode, or more than one mode.

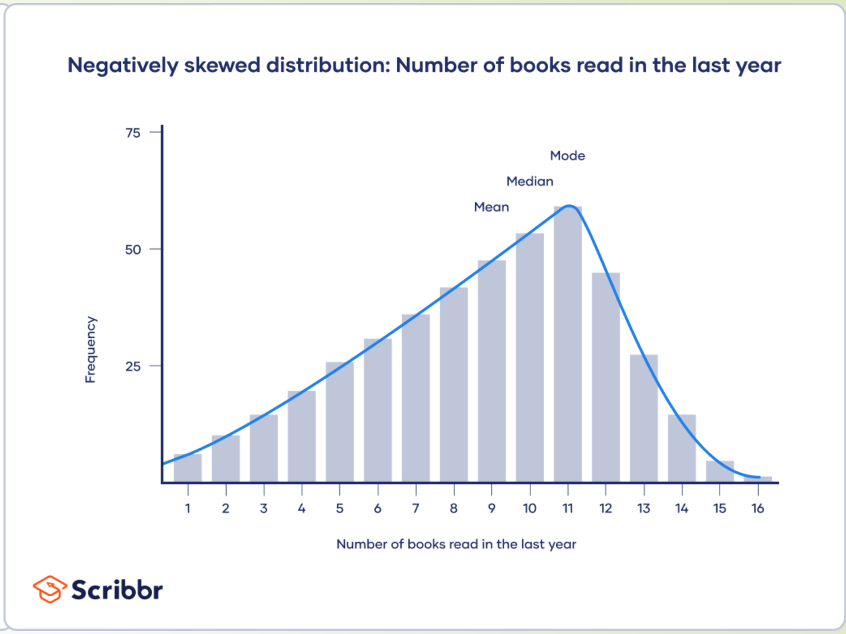
Examples: Consider the following data set – Mode is 7

Data (x)	7	10	15	20	25
Frequency (f)	5	2	2	3	4

Fundamentals of Measurements







Deviation & Standard Deviation

Deviation of a data is difference of the data from the mean value of a data set.

Standard deviation (σ) is defined as the root mean squared deviation.

Consider a data set: $x_1, x_2, x_3, \dots, x_n$

The mean value of the data set is

$$\mu = \frac{1}{n}(x_1 + x_2 + x_3, + \cdots + x_n) = \frac{1}{n}\sum_{i=1}^{n} x_i \equiv \sum_{i=1}^{n} \frac{n_i}{n} x_i = \sum_{i=1}^{n} p_i x_i$$

The deviations are $d_1 = x_1 - \mu, d_2 = x_2 - \mu, d_3 = x_3 - \mu, \dots, d_n = x_n - \mu$

Standard deviation

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} d_i^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (x_i - \mu)^2} \equiv \sqrt{\sum_{i=1}^{n} \frac{n_i}{n} (x_i - \mu)^2} = \sqrt{\sum_{i=1}^{n} p_i (x_i - \mu)^2}$$

p_i is the probability of getting the data x_i

Fundamentals of Measurements

Example

Serial No.	Measured diameter (mm)	Mean diameter (mm)	Deviation (mm)	Standard Deviation (mm)
1	1.97	2.00	-0.03	0.03
2	1.96		-0.04	
3	1.98		-0.02	
4	2.00		0	
5	2.01		0.01	
6	2.04		0.04	
7	1.99		-0.01	
8	2.03		0.03	
9	1.95		-0.05	
10	2.05		0.05	

Fundamentals of Measurements

Significant Figures

- Every measurement appears with certain degree of uncertainty – the errors.
- Results of measurement should be reported in a way that indicate the precision of measurement.
- Reported result of measurement is a number that includes *all digits in the number that are known reliably plus the first digit that is uncertain*.
- Significant digits or significant figures = reliable/certain digits + first uncertain digit

For example, the measurement of diameter of a wire with a screw gauge is reported as 2.07 mm. Here the first two digits 2 and 0 are certain/reliable while the third digit 7 is uncertain. So significant digits/figures are 3 here.

Fundamentals of Measurements

Significant Figures

- Significant figures indicate the precision of measurement which is determined by the least count of the measuring instrument.
- A choice of change of different units does not change the number of significant digits or figures in a measurement.

For example, the measurement of diameter of a wire with a screw gauge (least count 0.01 mm) is reported as 2.07 mm which has 3 significant digits. The result of measurement could be reported as 0.207 cm or 0.00207 m or 2070 μm. All have same significant digits 2, 0, 7 i.e. three in number.

Fundamentals of Measurements

Significant Figures

Contd. 0.207 cm = 0.00207 m = 2070 μm. Note that:

- All the non-zero digits are significant (for example, 2 and 7 above).
- All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all (for example, 0 between 2 and 7 above).
- If the number is less than 1, the zero(s) on the right of decimal point but to the left of the first non-zero digit are not significant (for example, first two zeros in 0.00207 are not significant).
- The trailing zero(s) in a number with a decimal point are significant (four example, the trailing zero in 1.350 is significant and hence the number of significant digits are 4; similarly, the trailing zeros in 0.03100 is significant and hence the number of significant digits are 4).

Fundamentals of Measurements

Significant Figures

- The terminal or trailing zero(s) in a number without a decimal point are not significant (for example, 123 m = 12300 cm = 123000 mm has three significant figures, the trailing zero(s) being not significant.

Measured length of an object is reported as 4.700 m. There are two trailing zeros after the decimal points and hence the number has 4 significant digits.

However, the measurement could be reported as 4700 mm. Here the trailing zeros are in a number with no decimal. So how many significant digits are there? 2 or 4?

Fundamentals of Measurements

Significant Figures

Contd.

Way Out: Report every measurement in scientific notation (in the power of 10). Every number in this notation is expressed in the form $a \times 10^b$ with the base number a between 1 and 10, and the exponent b is an integer. The exponent b is irrelevant to the determination of significant figures. However, all zeroes appearing in the base number in the scientific notation are significant.

For example, measured length of an object is reported in scientific notation as 4.700 m = 4.700×10^2 cm = 4.700×10^3 mm = 4.700×10^{-3} km. Note that the base number has 4 significant digits including the trailing zeros and no confusion arises in this notation.

Fundamentals of Measurements

Significant Figures: Rules for Arithmetic Operations

How to add the lengths 17.13 cm, 110.1 cm? or, how to subtract 0.304 m from 1.307 m?

Rule 1: In addition or subtraction, the final result should retain as many decimal places as are there in the number with the least decimal places.

Thus, 17.13 cm (correct upto two decimal places) + 110.1 cm (correct upto one decimal place) = 127.2 cm (correct upto one decimal place).

Similarly, 1.307 m – 0.304 m = 1.003 (all correct upto three decimal places)

As an another example, 436.37 g + 227.2 g + 0.301 g = 663.9 g (rounded off to first decimal place)

Fundamentals of Measurements

Significant Figures: Rules for Arithmetic Operations

The measured mass and volume of an object are reported as 4.237 g (4 significant digits) and 2.51 cc (3 significant digits). What is the density of the object?

Rule 2: In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures.

Thus, the density of the object = 4.237 g / 2.51 cc = 1.69 g/cc

Similarly, area of a sheet of length 5.1 cm and breadth 1.3 cm is 6.6 cm^2.

Problem: How many significant figures should be there in the light year, given the speed of light is 3.00×10^8 m/s and 1 year = 365.25 days?

Fundamentals of Measurements

Rounding off Uncertain Digits

Case 1: The preceding digit is raised by 1 if the insignificant digit to be dropped is more than 5, and is left unchanged if the latter is less than 5.

Examples: 1.746 rounded off to three significant figures is 1.75

1.743 rounded of to three significant figures is 1.74

Case 2: The insignificant digit is 5 – if the preceding digit is even, the insignificant digit is simply dropped and, if the preceding digit is odd, it is raised by 1.

Examples: 1.745 rounded off to three significant figures is 1.74

1.735 rounded off to three significant figures is 1.74

Fundamentals of Measurements

Order of Magnitude

- The term order of magnitude is important when an estimation is required.
- Represent the number in scientific notation: $a\times10^{b}$ with the base a rounded off to 1 for $a \leq 5$, and to 10 for $a \geq 5$. Thus the number can be approximately read as 10^{b} in which the exponent b is called the order of magnitude.
- Examples: Diameter of earth (1.28×10^{7} m) is of the order of 10^{7} m with the order of magnitude 7. Diameter of H-atom (1.06×10^{-10} m) is of the order of 10^{-10} m, with the order of magnitude –10. Thus, the diameter of the earth is 17 orders of magnitude larger than the hydrogen atom.

Fundamentals of Measurements

Errors

- ❑ Doesn't mean mistake or blunder.
- ❑ Uncertainties in scientific measurements are inevitable – errors as uncertainties in measurement.
- ❑ Errors cannot be eliminated, but reduced.
- ❑ Errors are broadly classified into two: (a) **Random Errors** (b) **Systematic Errors**

Fundamentals of Measurements

(a) Random Errors

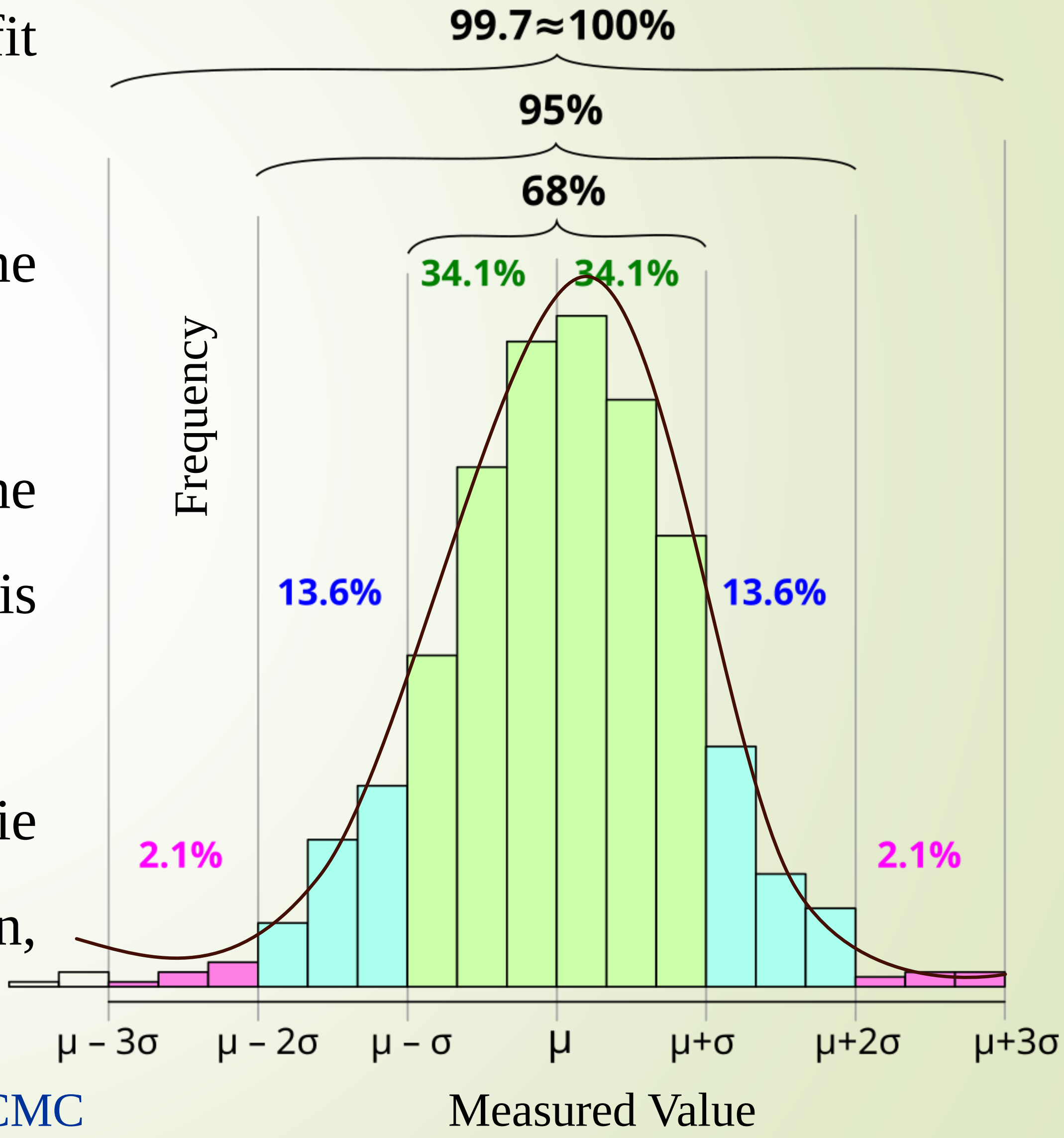
- Repeated measurements of a physical quantity does not produce identical result.
- Random errors in experimental measurements are caused by unknown and unpredictable changes in the experiment.
- These changes may occur in the measuring instruments or in the environmental conditions.
- For examples, random errors appear due to electronic noise in the circuit of an electrical instrument; irregular changes in the heat loss rate from a solar collector due to changes in the wind.
- Random errors can be minimized by taking a large number of careful measurements of the same physical quantity on an identically prepared system or a number of identical systems (ensemble).

https://www.physics.umd.edu/courses/Phys276/Hill/Information/Notes/ErrorAnalysis.html

Fundamentals of Measurements

(a) Random Errors

- Large number of identical measurements generally fit to normal/gaussian distribution.
- The mean μ of a number of measurements of the same quantity is the best estimate of that quantity.
- Standard deviation (σ) determines the precision of the measurements. Lower the standard deviation, better is the precision of the measurements.
- Approximately 68%, 95%, and 99.7% of the values lie within 1σ, 2σ and 3σ deviations of the mean, respectively.



Fundamentals of Measurements

(b) Systematic Errors

- Systematic errors are predictable variations in measurements that cause observations to differ from the intended measurement.
- These errors can occur when all measurements of a physical quantity are affected equally, resulting in consistent differences in readings.
- Systematic errors are also known as ‘statistical bias’ because they skew data in standardized ways that hide true values and can lead to inaccurate conclusions.
- Systematic errors can be caused by incorrectly used equipment, poor calibration, instrument calibration failure, data handling, and offset.

Fundamentals of Measurements

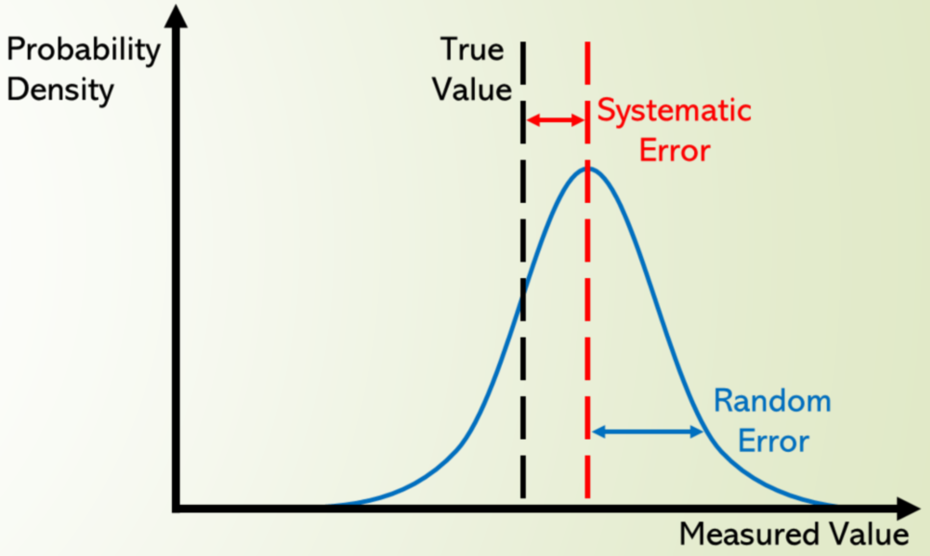
(b) Systematic Errors

- Examples: Zero error in screw gauge, slide caliper; end error in meter bridge experiment, offset voltage of OPAMP etc.
- In some cases the systematic errors can be determined and eliminated accordingly.
- Accuracy of a measurement is determined by the systematic errors. If an experiment has smaller systematic errors, it is said to have higher accuracy.

Fundamentals of Measurements

Error Analysis

- Error in observational or measurement is the difference between a measured value of a quantity and its unknown true value or actual value.
- What is true value of a measurement? It is vaguely defined since every measurement appears with errors.
- Error analysis refers to the standard procedure of finding out by what margin the measured value differ from the true value even without the information of the later.



 https://en.wikipedia.org/wiki/Observational_error

Fundamentals of Measurements

Error Analysis

- True value of a measurement can not be defined. However, error analysis helps to find out the range within which the true value is likely to be.
- Relative or proportional error (or the percentage error) bears the greater importance in error analysis rather than the absolute error.
- Maximum proportional error can be calculated in order to determine the limit of accuracy of a measurement.

Error Analysis: Maximum Proportional Error

❑ **Example 1:** Suppose to measure the length and breadth of a rectangular sheet with a slide caliper (vernier constant = 0.01 cm) as 2.06 cm and 1.01 cm respectively. What is the perimeter of the sheet?

Perimeter $S = 2(l + b)$; length $l = 2.06$ cm, breadth $b = 1.01$ cm and hence $S = 6.14$ cm.

Now the error in measuring S is $\Delta S = 2(\Delta l + \Delta b)$ and hence maximum proportional error

$$\frac{\Delta S}{S} = \frac{2(\Delta l + \Delta b)}{S} = \frac{2(0.01 + 0.01)}{6.14} = 0.01 \equiv 1\%$$

Note that, Δl and Δb are taken as 0.01 cm each since the length and the breadth are measure with the slide caliper with vernier constant (smallest division) 0.01 cm.

Error Analysis: Maximum Proportional Error

- **Example 2:** Suppose to measure the length and breadth of a rectangular sheet with a slide caliper (vernier constant = 0.01 cm) as 2.06 cm and 1.01 cm respectively. How long is the length relative to breadth?

$\ell = (l - b)$; length $l = 2.06$ cm, breadth $b = 1.01$ cm and hence $\ell = 1.05$ cm.

Now the error in measuring ℓ is $\Delta\ell = \Delta l + \Delta b$ and hence maximum proportional error

$$\frac{\Delta\ell}{\ell} = \frac{(\Delta l + \Delta b)}{\ell} = \frac{(0.01 + 0.01)}{1.05} = 0.02 \equiv 2\%$$

Note that, errors in measuring Δl and Δb (each 0.01 cm) are added to each other though $\ell = l - b$ since we are interested in determining maximum proportional error.

Error Analysis: Maximum Proportional Error

- **Example 3:** Suppose to measure the length and breadth of a rectangular sheet with a slide caliper (vernier constant = 0.01 cm) as 2.06 cm and 1.01 cm respectively. Find the area of the sheet.

Area $A = l \times b$; length $l = 2.06$ cm, breadth $b = 1.01$ cm and hence $A = 2.08\ \text{cm}^2$.

Now the error in measuring A is $\Delta A = \Delta l \times b + l \times \Delta b$ and hence maximum proportional error

$$\frac{\Delta A}{A} = \frac{\Delta l \times b + l \times \Delta b}{l \times b} = \frac{\Delta l}{l} + \frac{\Delta b}{b} = \frac{0.01}{2.06} + \frac{0.01}{1.01} = 0.01 \equiv 1\%$$

Error Analysis: Maximum Proportional Error

- **Example 4:** A physical quantity u is measured by measuring three quantities x, y and z with the working formula $u = x^{\alpha} y^{\beta} z^{-\gamma}$ where α, β and γ are positive constants.

$$\ln u = \alpha \ln x + \beta \ln y - \gamma \ln z$$

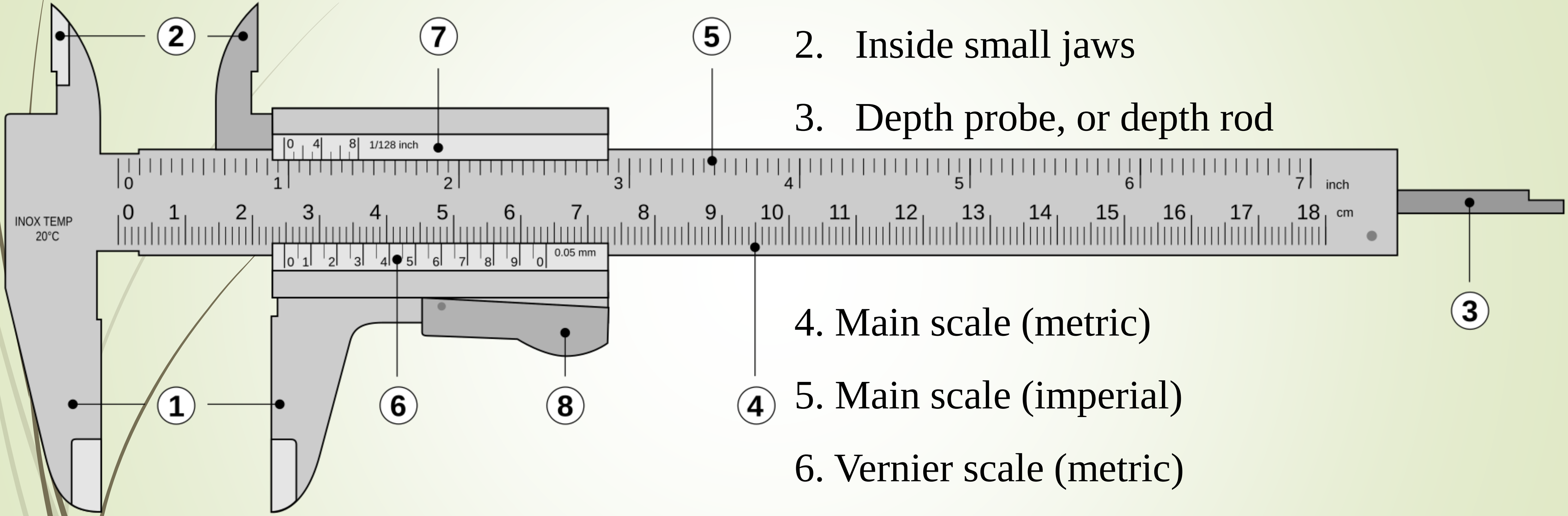
Proportional error

$$\frac{\Delta u}{u} = \alpha \frac{\Delta x}{x} + \beta \frac{\Delta y}{y} - \gamma \frac{\Delta z}{z}$$

Maximum proportional error

$$\left(\frac{\Delta u}{u}\right)_{\max} = \alpha \frac{\Delta x}{x} + \beta \frac{\Delta y}{y} + \gamma \frac{\Delta z}{z}$$

Slide Calipers



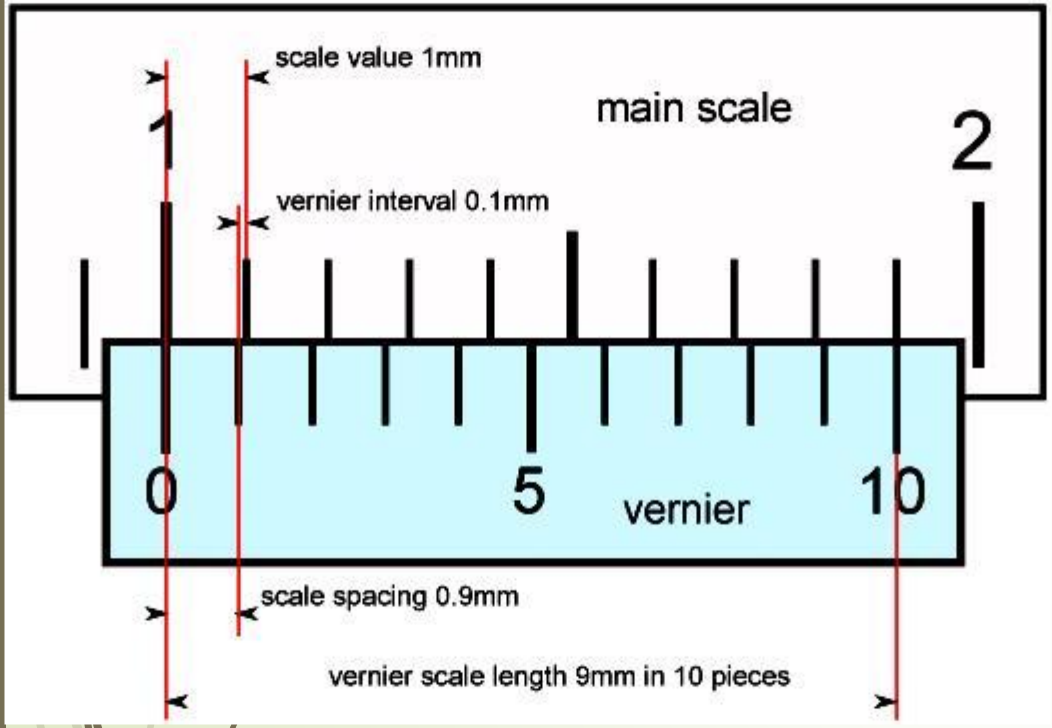
1. Outside large jaws
2. Inside small jaws
3. Depth probe, or depth rod

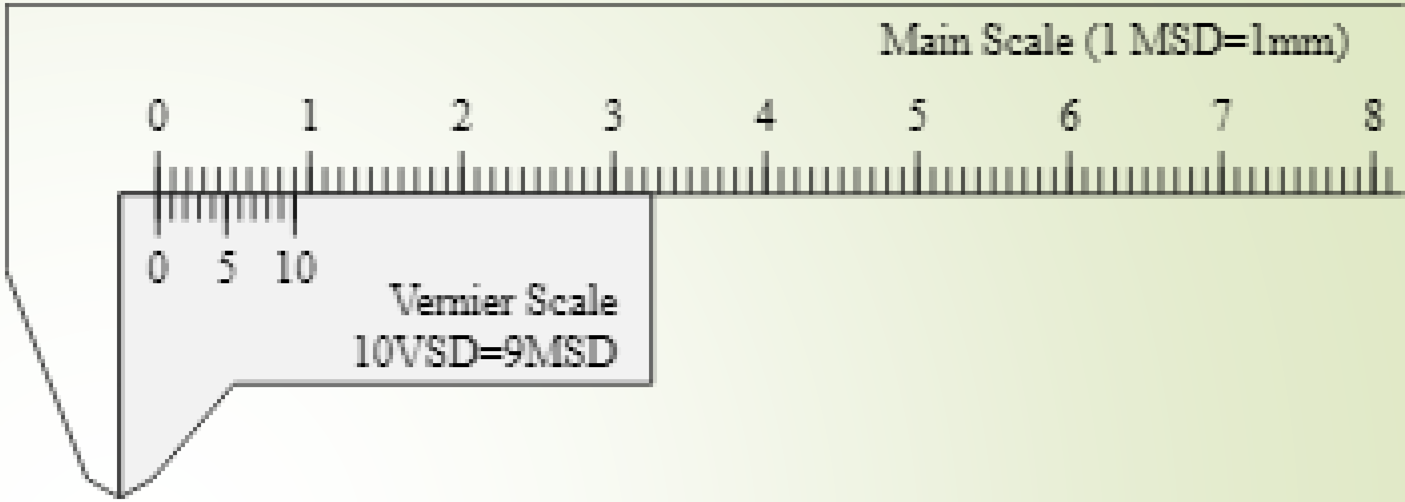
4. Main scale (metric)
5. Main scale (imperial)
6. Vernier scale (metric)
7. Vernier scale (imperial)
8. Retainer

https://en.wikipedia.org/wiki/Calipers

Slide Calipers

Vernier Constant:





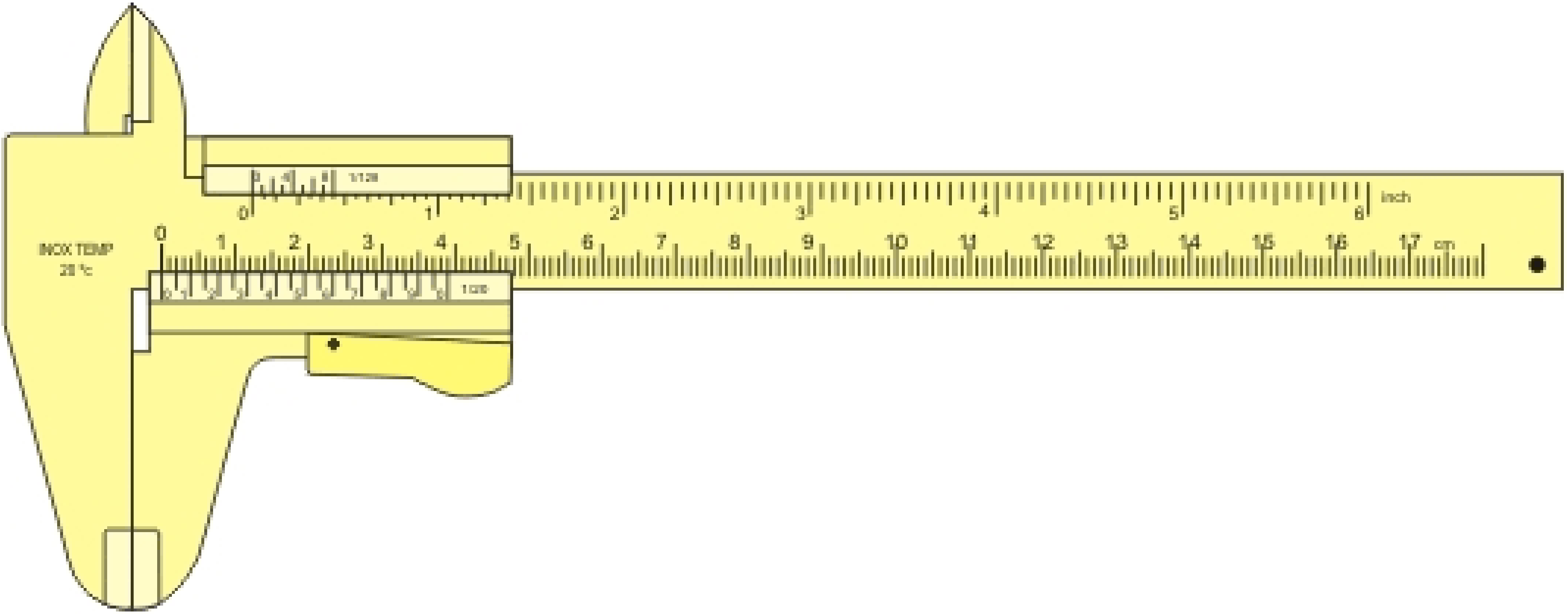
Vernier Constant (VC) / Least Count (LC) = Main Scale Division (MSD) – Vernier Scale Division (VSD). It is the smallest length that can be measured accurately with a Vernier calipers. For the given Vernier calipers

MSD = 1 mm, VSD = (9/10) MSD = 0.9 mm

Hence, VC = 0.1 mm = 0.01 cm

Slide Calipers

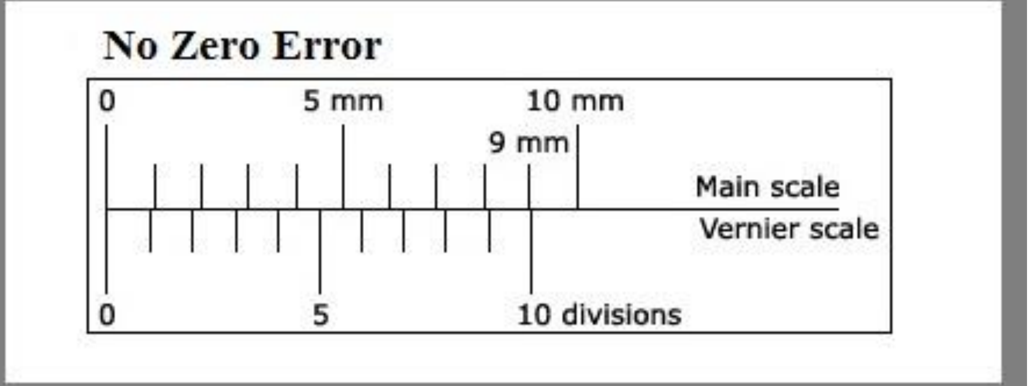
How to Measure: x = Main scale reading + Vernier scale reading × Vernier Constant

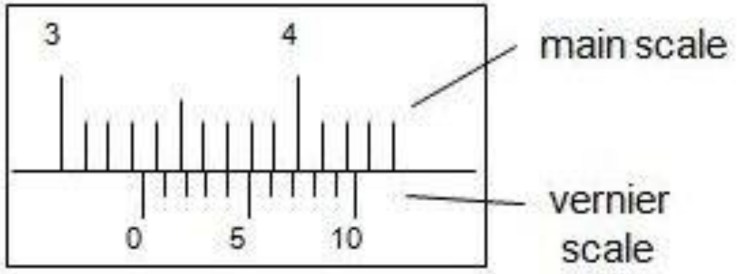


https://en.wikipedia.org/wiki/Calipers

Slide Calipers

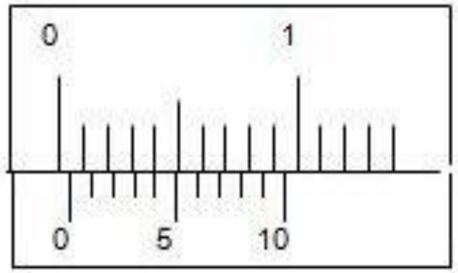
Zero Error: x = Main scale reading + Vernier scale reading × Vernier Constant – zero error





measured reading = 3.3 + 0.04 = 3.34

fig. (a)



Vernier scale zero is left of main scale zero

Correct measurement with positive error = 3.34 - 0.04 = 3.30

positive zero error = 0.04

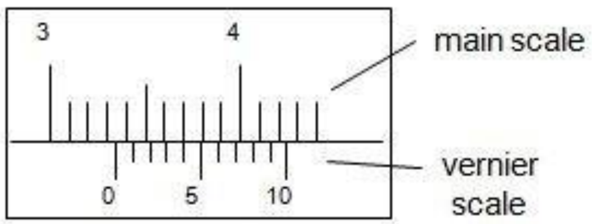
fig. (b)

Positive Error: When the two jaws are in contact and the zero of the vernier lies right to the zero of the main scale, the error is positive and the zero correction is negative.

Slide Calipers

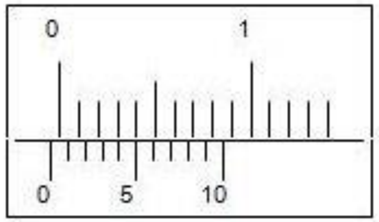
Zero Error: x = Main scale reading + Vernier scale reading × Vernier Constant – zero error

Negative Error: When the two jaws are in contact and the zero of the vernier lies left to the zero of the main scale, then the error is negative and the zero correction is positive.



measured reading = 3.3 + 0.04 = 3.34

fig. (a)



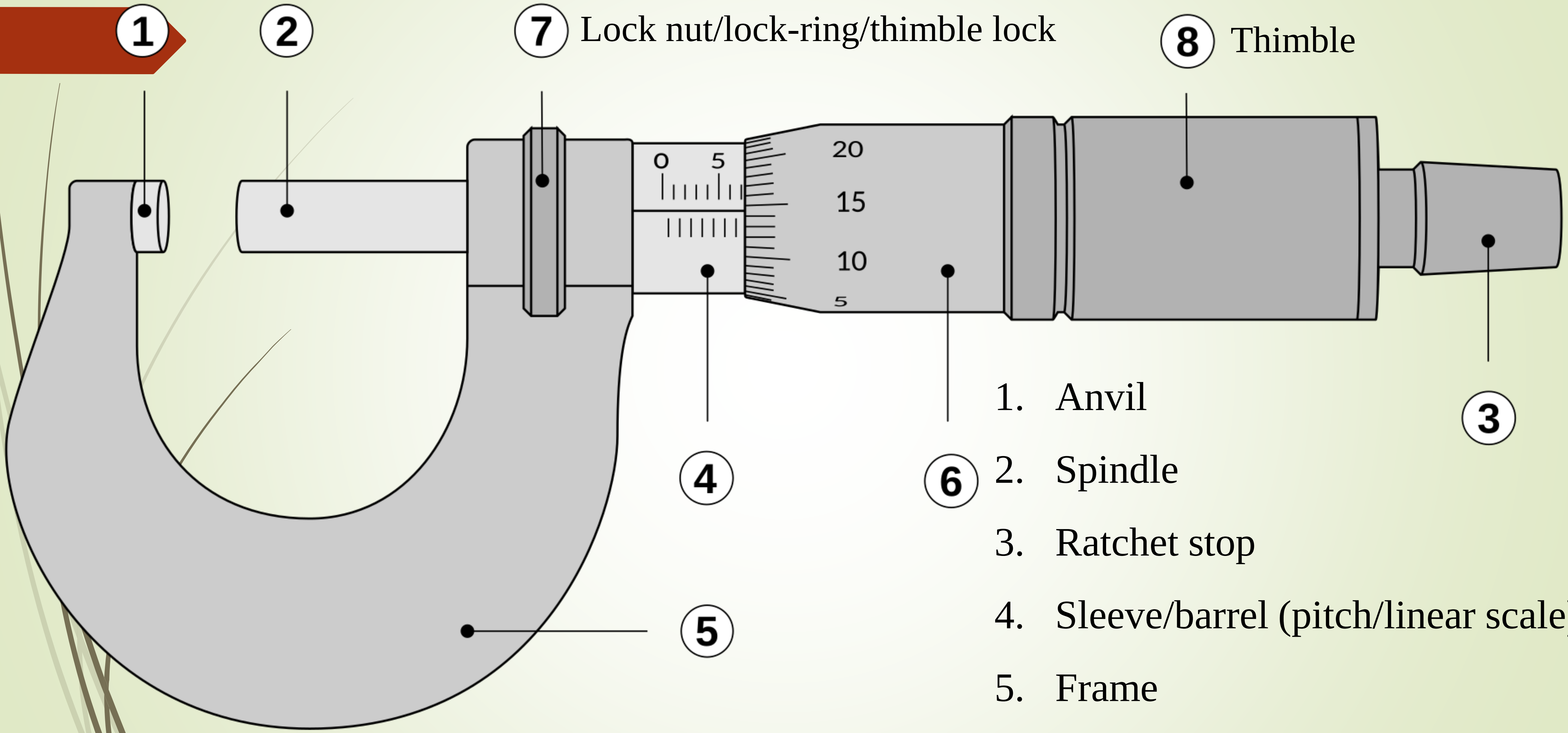
Vernier scale zero is right of main scale zero

Correct measurement with negative error = 3.34 - (-0.04) = 3.38

negative error = - 0.04

fig. (c)

Screw Gauge



1. Anvil
2. Spindle
3. Ratchet stop
4. Sleeve/barrel (pitch/linear scale)
5. Frame
6. Thimble scale

https://en.wikipedia.org/wiki/Micrometer_(device)

Screw Gauge

Pitch & Least Count:

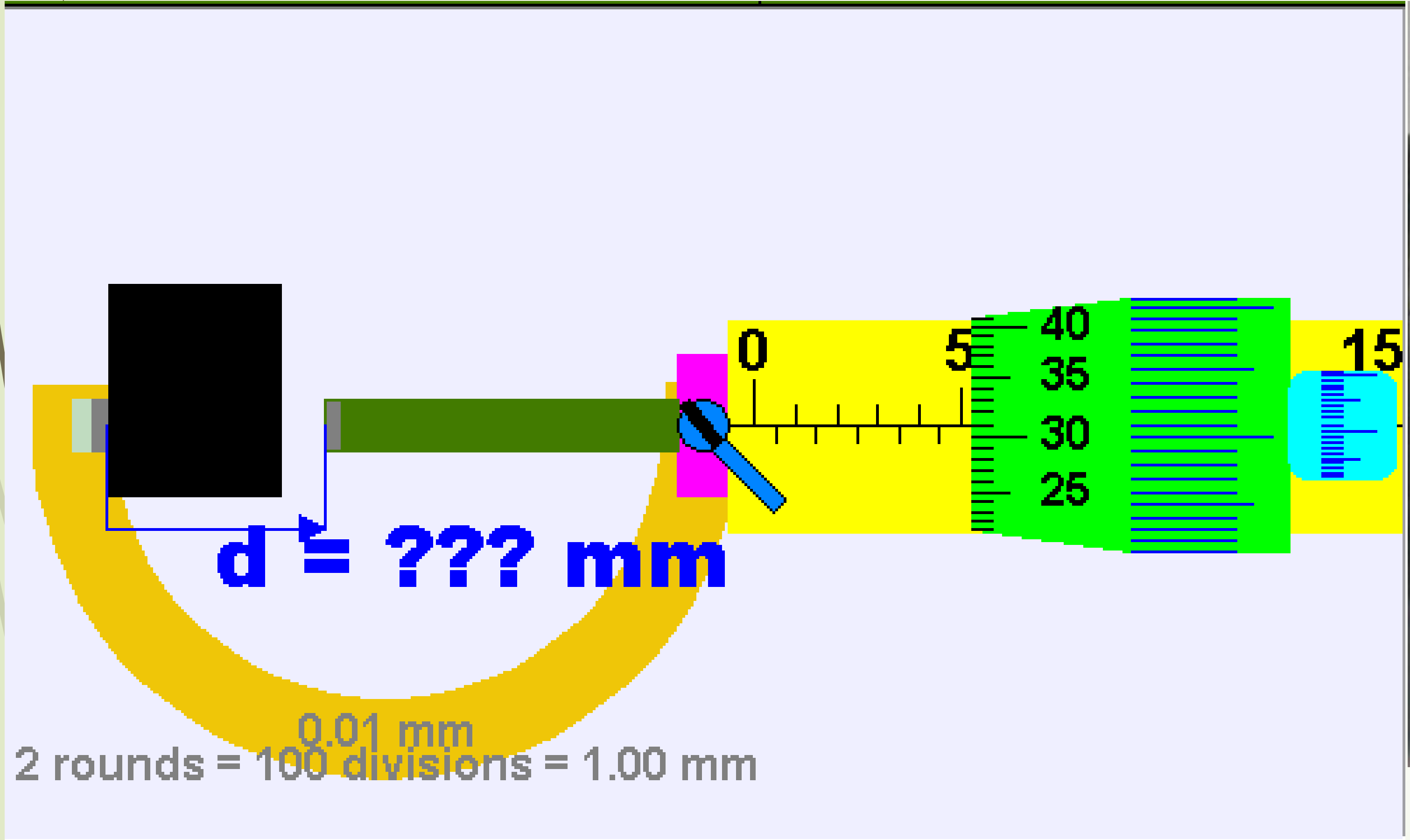
- **Pitch:** The amount of rotation of an accurately made screw can be directly and precisely correlated to a certain amount of axial movement (and vice versa), through the constant known as the screw's lead or pitch. A screw's lead or pitch is the distance it moves forward axially with one complete turn (360°).
- **Least Count** = Screw pitch/total number of divisions in thimble scale (circular scale)
- **Examples:** Axial movement of 5 mm on complete 5 rotations of the barrel (circular scale); circular scale has total 100 divisions

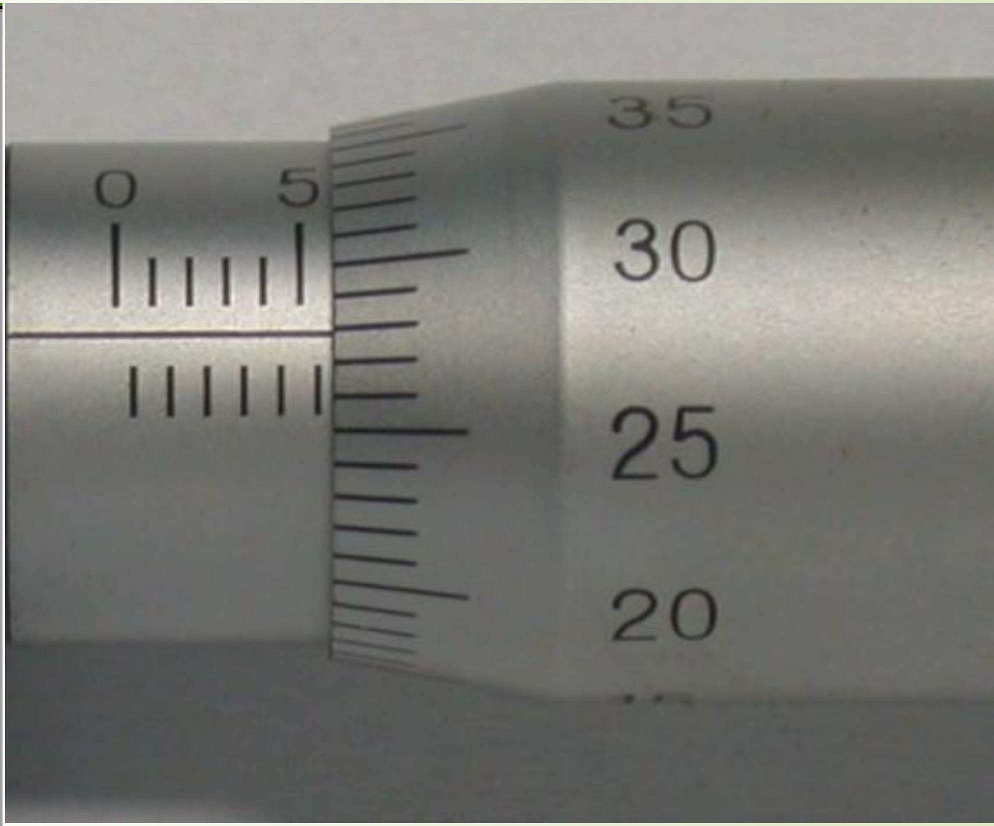
  Pitch = linear shift/number of complete rotations = 1 mm

  LC = 1/100 mm = 0.01 mm

Screw Gauge

How to Measure: x = Linear scale reading + Circular scale reading × Least Count





LC = 0.01 mm

Linear Scale Reading = 5.5 mm

Circular Scale Reading = 28

Measured dimension = (5.5 + 28 × 0.01) mm

= 5.78 mm

Screw Gauge

Zero Error: Corrected reading = measured reading – zero error

- There is no error when the jaws are closed and the zeros of both the circular scale and the linear scale exactly overlap [figure (a)].
- Positive Zero Error: If the circular scale's zero is below the reference or index line, the zero error will be positive. So, zero correction will be positive. [figure (b)]
- Negative Zero Error: The error is negative if the zero of the circular scale is above the index line (reference) line. So, zero correction will be positive. [figure (c)]