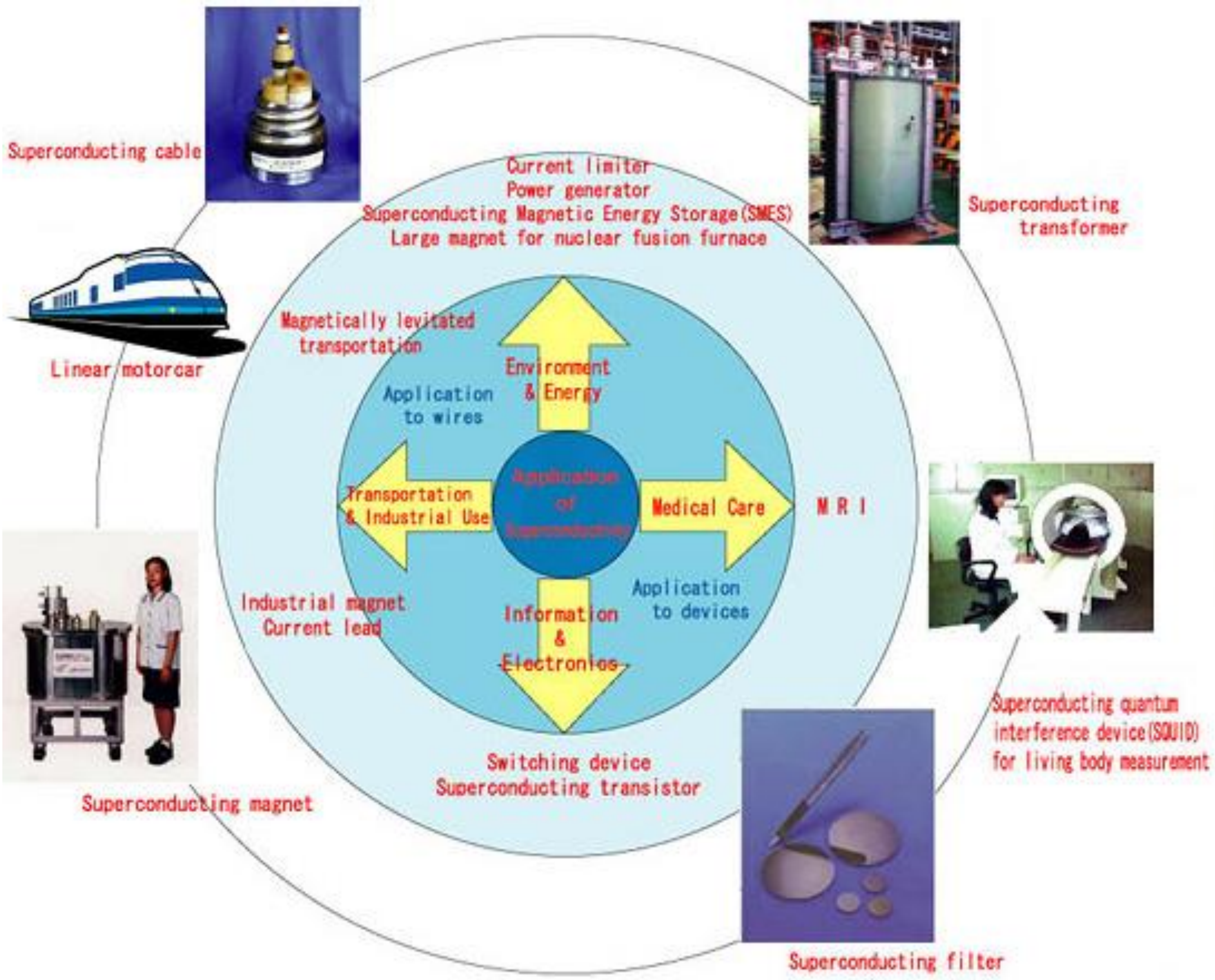
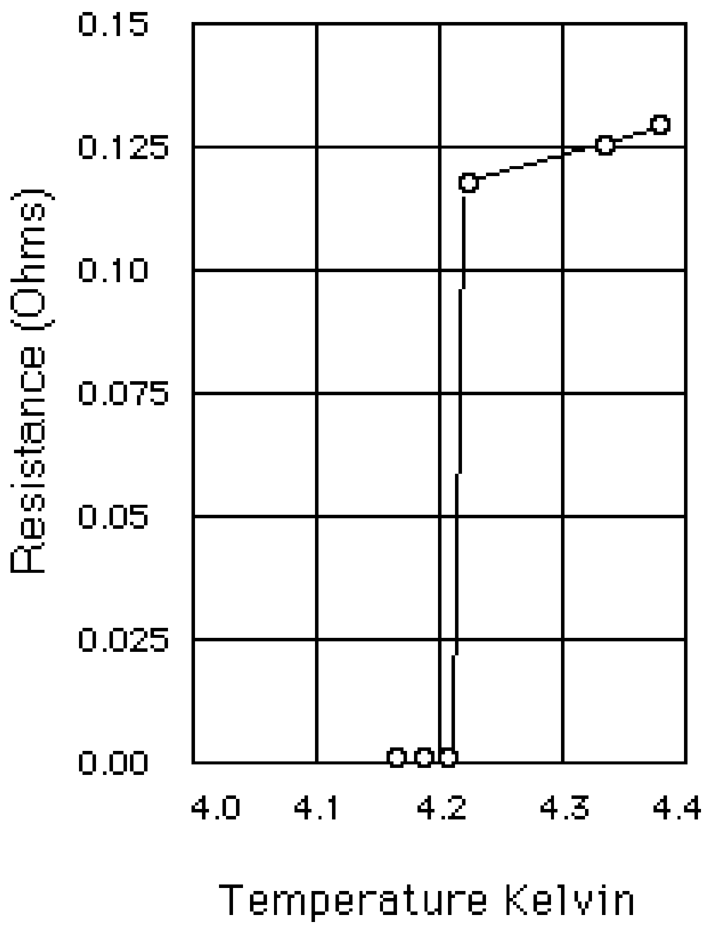
Superconductivity

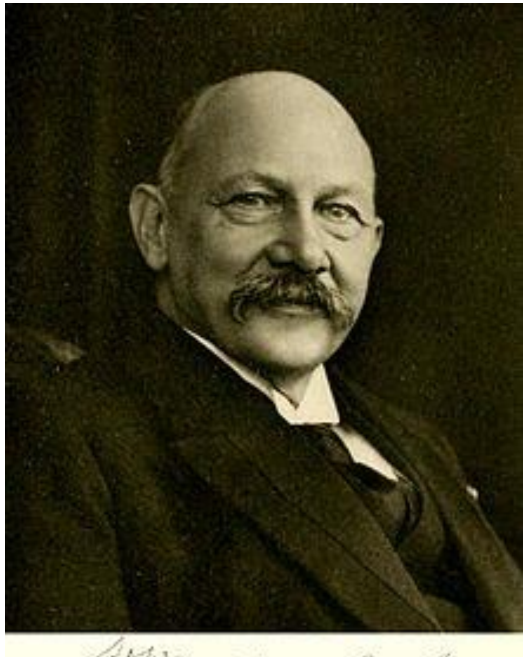
Syllabus: Experimental Results. Critical Temperature. Critical magnetic field. Meissner effect. Type I and type II Superconductors, London's Equation and Penetration Depth. Isotope effect.



Discovery

- Discovered in 1911 by Kamerlingh Onnes while measuring the resistivity of solid mercury at cryogenic temperature using the recently-discovered (by himself) liquid helium as refrigerant
- Onnes observed the resistivity abruptly disappeared at temperature of 4.2 K





Nobel Prize in Physics, 1913

Properties

Properties not affected:

- Elastic properties
- Thermal expansion behaviour
- Photoelectric properties
- Internal arrangement of crystal lattice as confirmed by X-ray diffraction pattern before and after such a transition

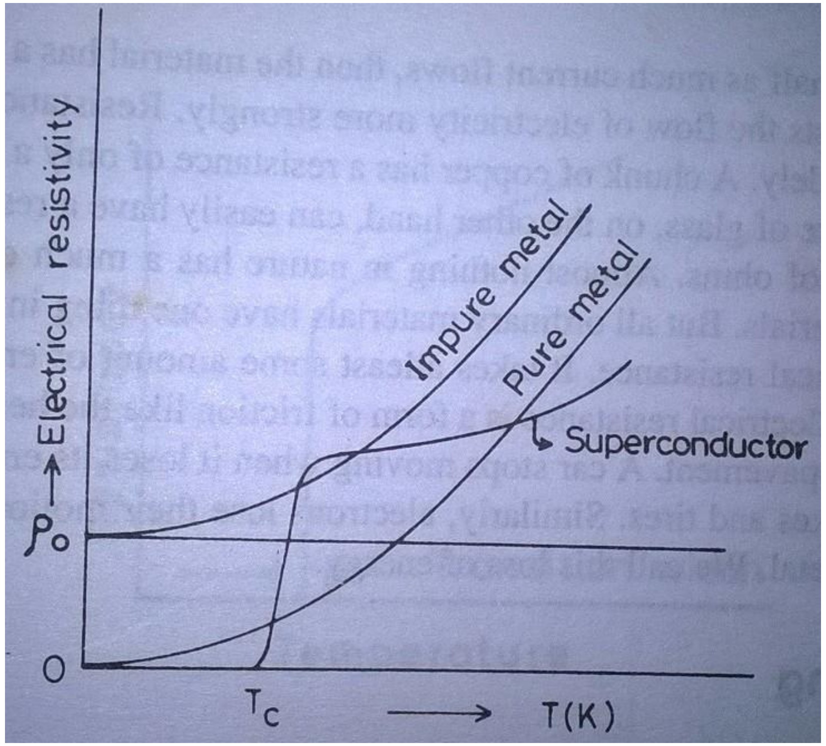
Properties

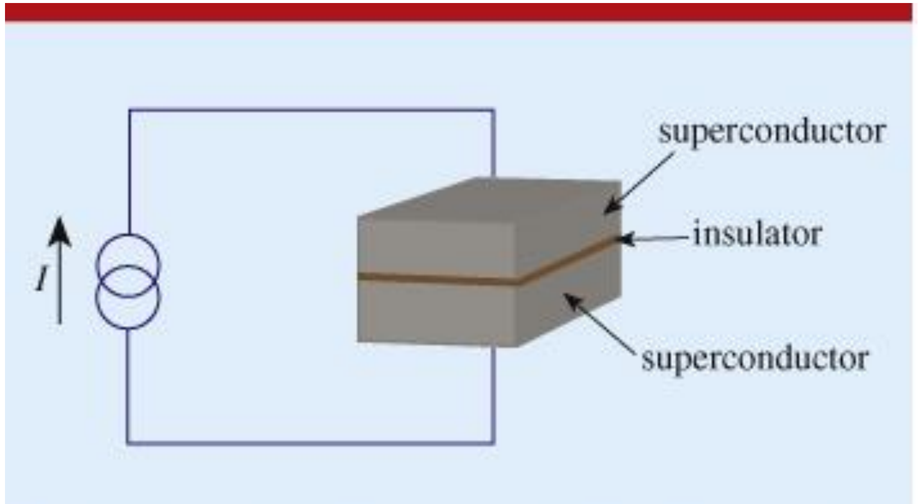
Properties affected:

- Magnetic properties change
- Electrical properties – electrical resistivity tends to zero at $T=T_c$
- All thermo-electric effects disappear for $T \leq T_c$
- Specific heat shows a discontinuous change
- Entropy shows a decrease for $T \leq T_c$

Electrical Properties

- Below a critical temperature T_c, the resistance of superconducting materials becomes almost zero
- Current flows indefinitely with no power loss
- No potential difference is required to maintain current
- A ‘super-current’ can flow across an insulating junction – **Josephson Effect**

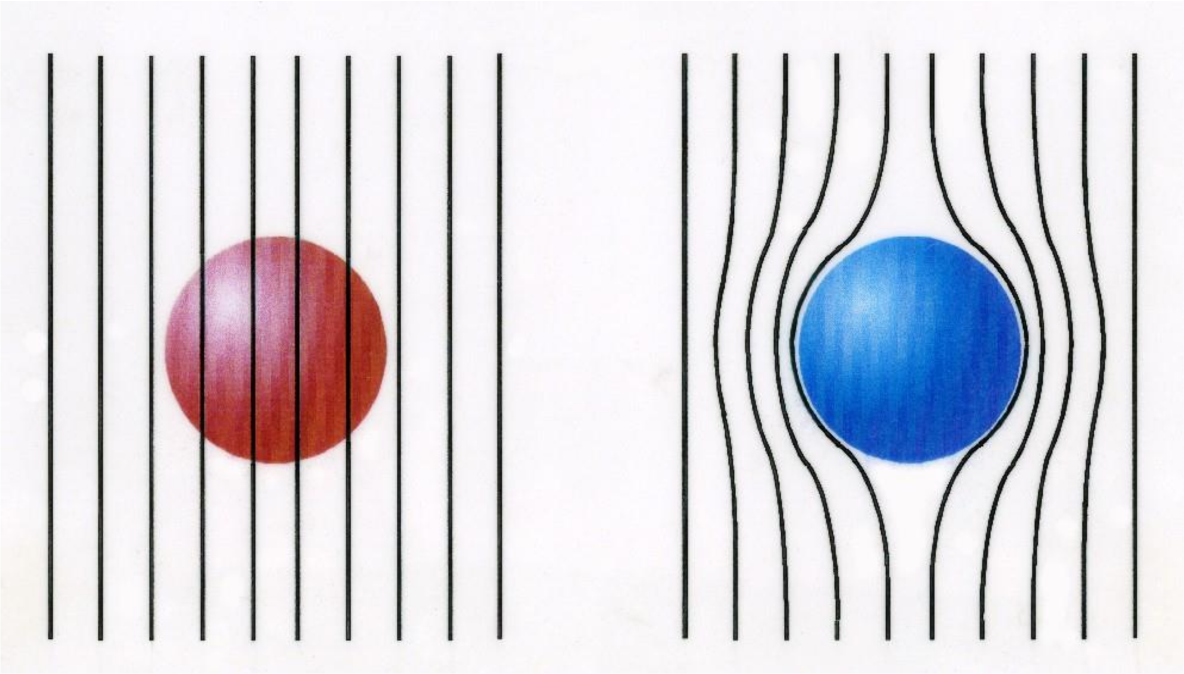




Effect of Magnetic Field

- Below the critical temperature T_c & below the critical magnetic field H_c, superconducting materials expel magnetic flux through it i.e., behave as perfect diamagnet ($M = -H$, susceptibility $\chi = -1$) – **Meissner Effect**

$T > T_c$ ***$T < T_c$***



- This 'diamagnetic' property is more fundamental than the zero resistance property for the superconducting materials

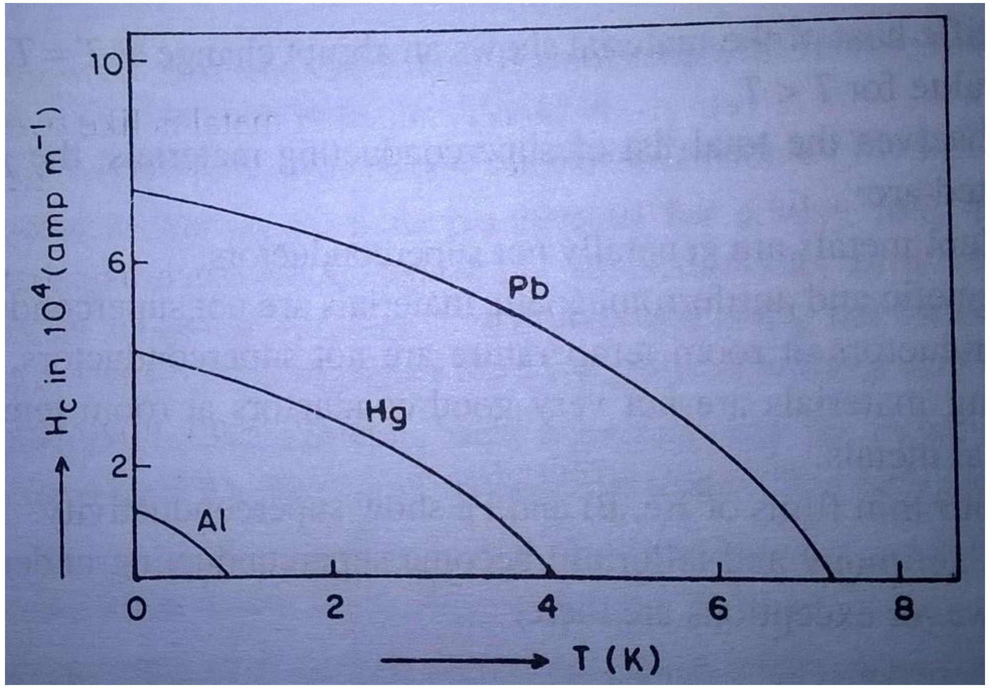
Meissner Effect

- Such a strong diamagnetic property can cause the material to levitate in space – Superconducting Magnetic Levitation

- The critical temperature T_c and the critical magnetic field H_c for superconducting materials are interdependent

$$H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

H_c = critical field strength at temperature T
H_0 = critical field strength at absolute zero temperature
T_c = critical temperature



Type-I and type-II superconductors

- The interior of a bulk superconductor cannot be penetrated by a weak magnetic field - Meissner effect
- For large magnetic field, above the critical value H_c, superconductivity breaks down
- Superconductors can be divided into two types according to how this breakdown occurs:

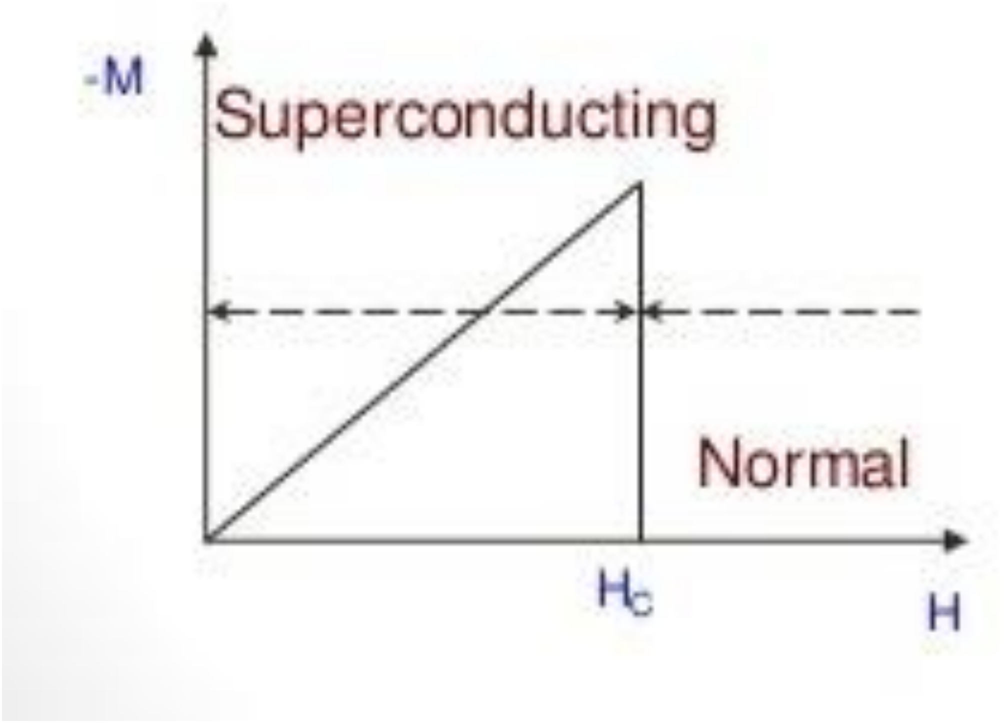
Type-I superconductors

&

Type-II superconductors

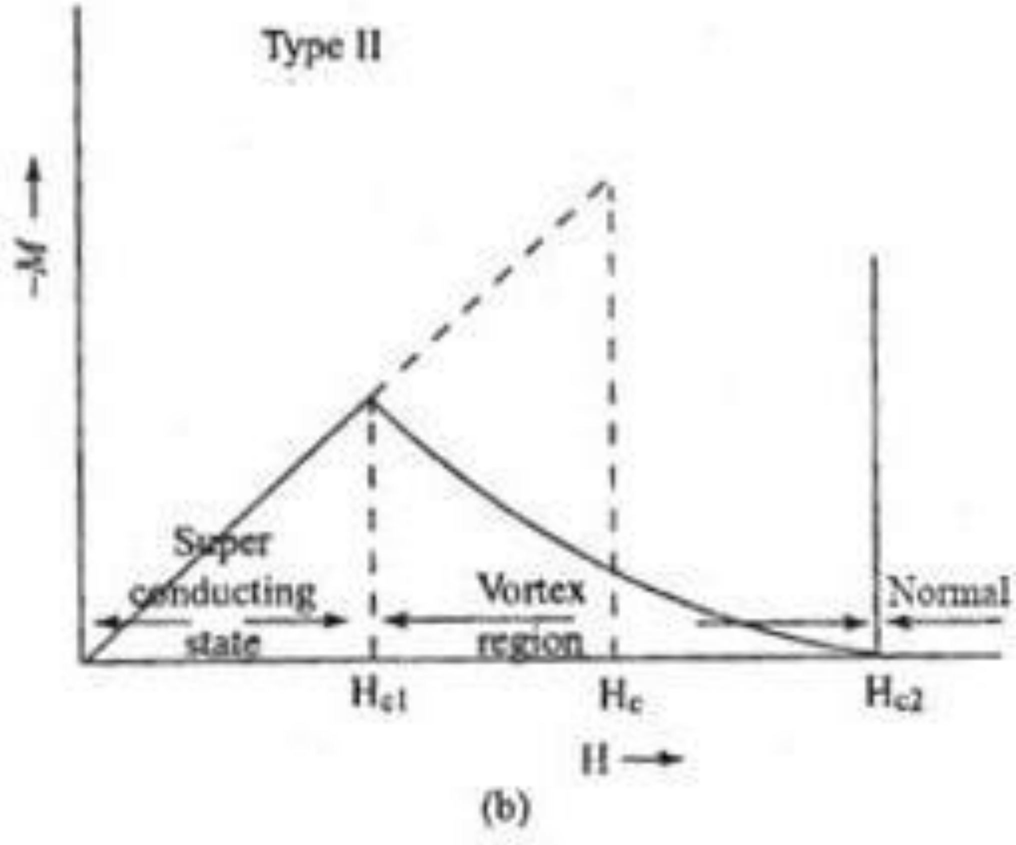
Type-I superconductors

- Expels all magnetic flux for $T<T_c$ and $H<H_c$
- Superconductivity abruptly destroyed at $H>H_c$ – 1st order phase transition
- Features generally exhibited by pure metals – Al, Pb, Hg

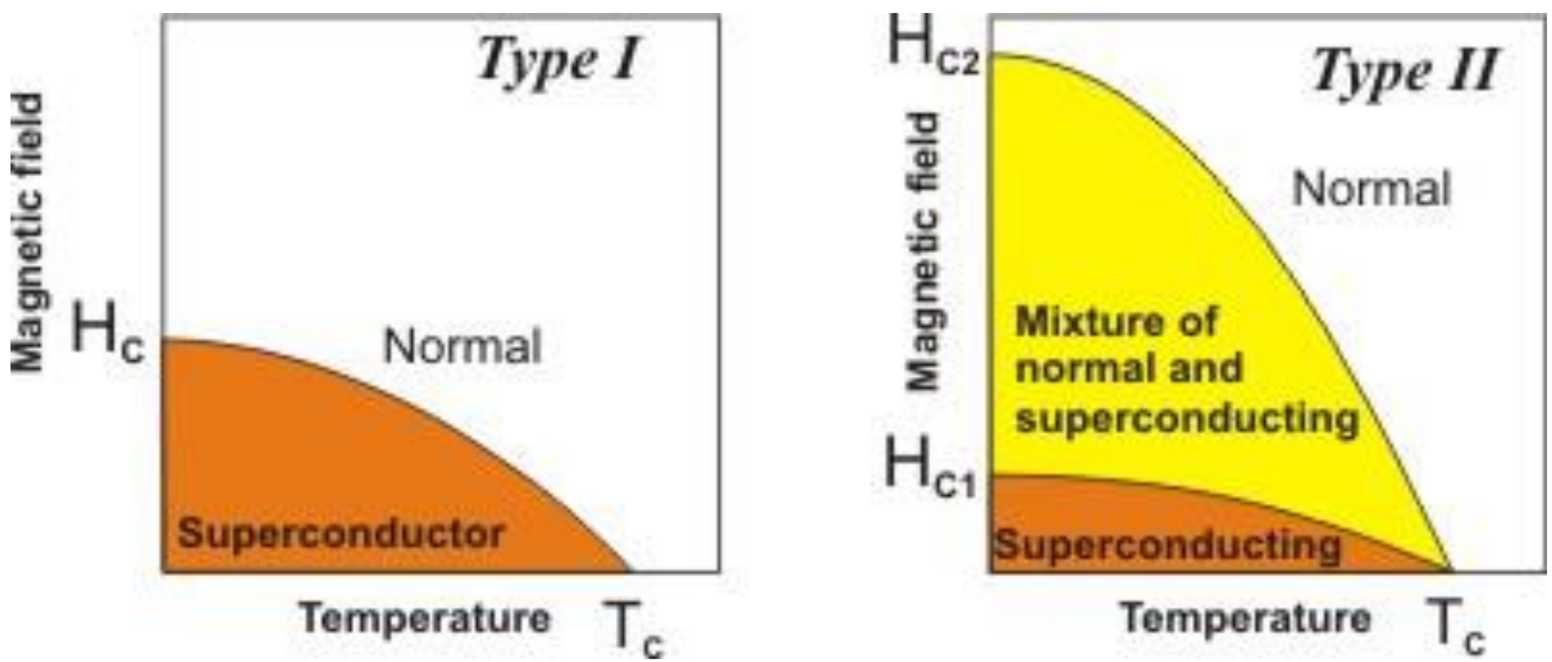


Type-II superconductors

- Two critical fields H_{c1} & H_{c2}
- Expels all magnetic flux for $T<T_c$ and $H<H_{c1}$
- For $H>H_{c1}$ magnetic flux is partially expelled, and the material enters a state of mixed normal and superconductivity (vortex region)
- Superconductivity disappears at $H>H_{c2}$
- Features generally exhibited by alloys – NbTi, YBCO etc



Type-I and type-II superconductors



$$H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

Penetration Depth

We have *Maxwell's electromagnetic equation*

$$\nabla \times \vec{B} = \mu_0 \vec{J}_s.$$

Taking curl on both sides,

$$\nabla \times \left(\nabla \times \vec{B}\right) = \mu_0 \left(\nabla \times \vec{J}_s\right) \text{ or, } \nabla\left(\nabla \cdot \vec{B}\right) - \nabla^2 \vec{B} = \mu_0 \left(\nabla \times \vec{J}_s\right)$$

$$\text{or, } -\nabla^2 \vec{B} = \mu_0 \left(\nabla \times \vec{J}_s\right) \quad \left[\because \nabla \cdot \vec{B} = 0\right]$$

$$\text{or, } \nabla^2 \vec{B} = -\frac{\mu_0 n_s e^2}{m} \vec{B} \quad \because \vec{\nabla} \times \vec{J}_s = -\frac{n_s e^2}{m} \vec{B}$$

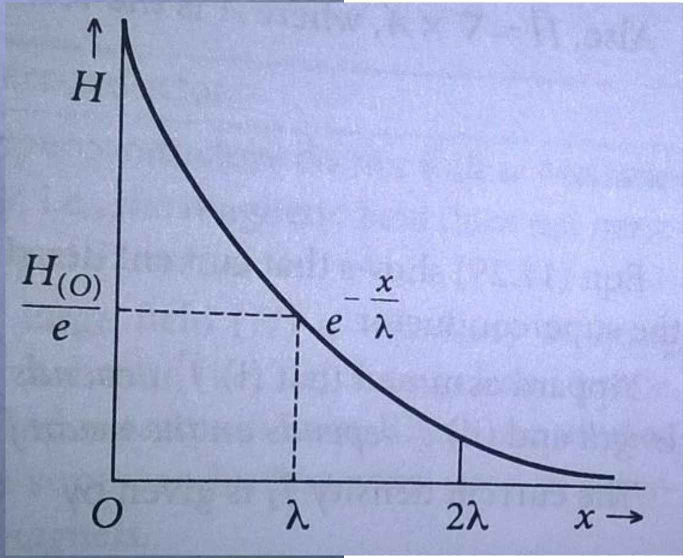
$$\text{or, } \nabla^2 \vec{B} = \frac{\vec{B}}{\lambda^2},$$

where $\lambda = \left(\frac{m}{\mu_0 n_s e^2}\right)^{1/2}$ is called *London penetration depth*.

The solution of eqn (11.22) is

$$H = H(0)\, e^{-\frac{x}{\lambda}},$$

where $\vec{B} = \mu_0 \vec{H}$.



Isotope Effect

- The critical transition temperature (T_c) shifts with the isotopic mass (M)

$$T_c \propto M^{-1/2}$$

- Larger the isotopic mass, lower is the critical transition temperature

The transition temperature of ordinary mercury of atomic mass 200.59 is 4.153 K, find out the transition temperature of mercury having atomic mass 204

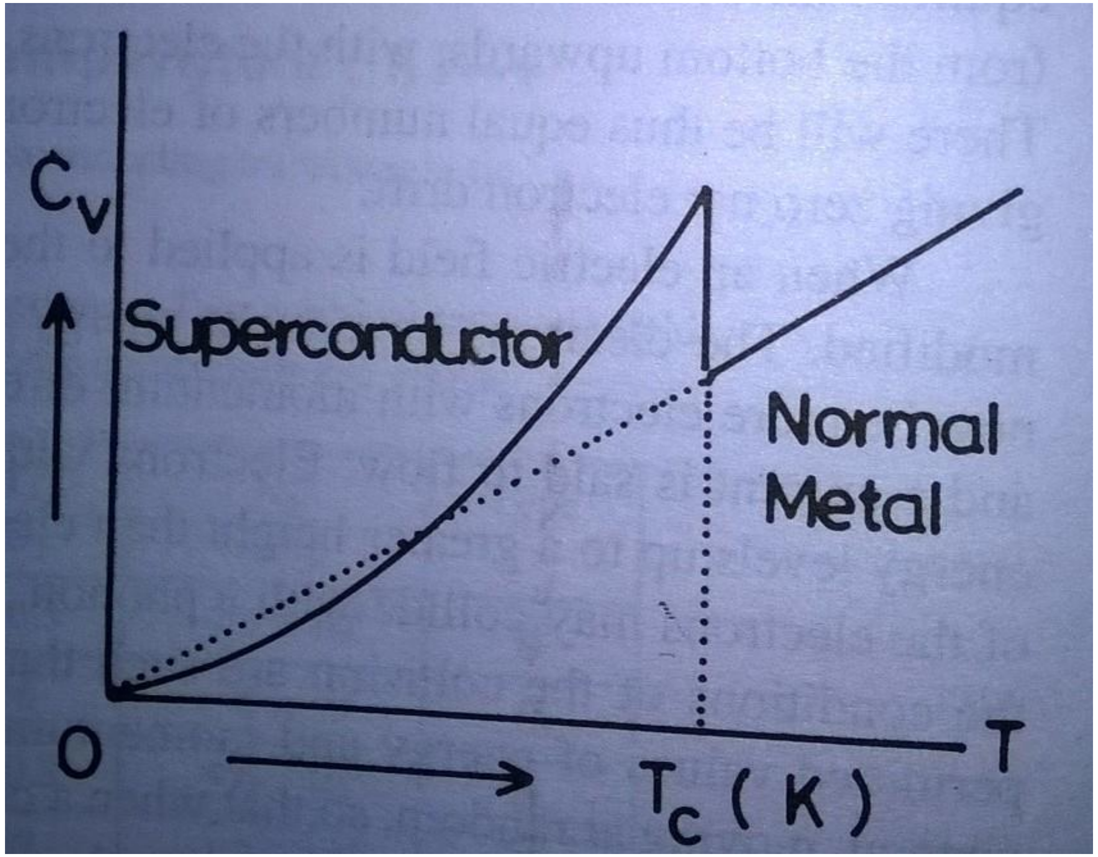
$$\frac{T_{c1}}{T_{c2}} = \sqrt{\frac{M_2}{M_1}}$$

$$\Rightarrow T_{c1} = T_{c2}\sqrt{\frac{M_2}{M_1}} = 4.153 \times \sqrt{\frac{200.59}{204}}\,\mathrm{K} = 4.118\,\mathrm{K}$$

$$\text{Actually, } T_c \propto M^{-\beta},$$
$$\beta = +0.5 \text{ or, some different value}$$

Thermal Properties

- Specific heat for normal metal: $C_v = aT + bT^3$ – first term due to specific heat of electrons in the metal & the second term due to lattice vibrations (at low temperature)
- For superconductors C_v shows a jump at T_c
- C_v falls almost exponentially as T decreases below T_c in superconducting state



Thermal Properties

- This exponential behaviour is an indication of the existence of a finite gap in the energy spectrum of electrons separating the ground state from the lowest state

$$C_{sc}(T) = A\exp(-E_g / 2k_B T)$$

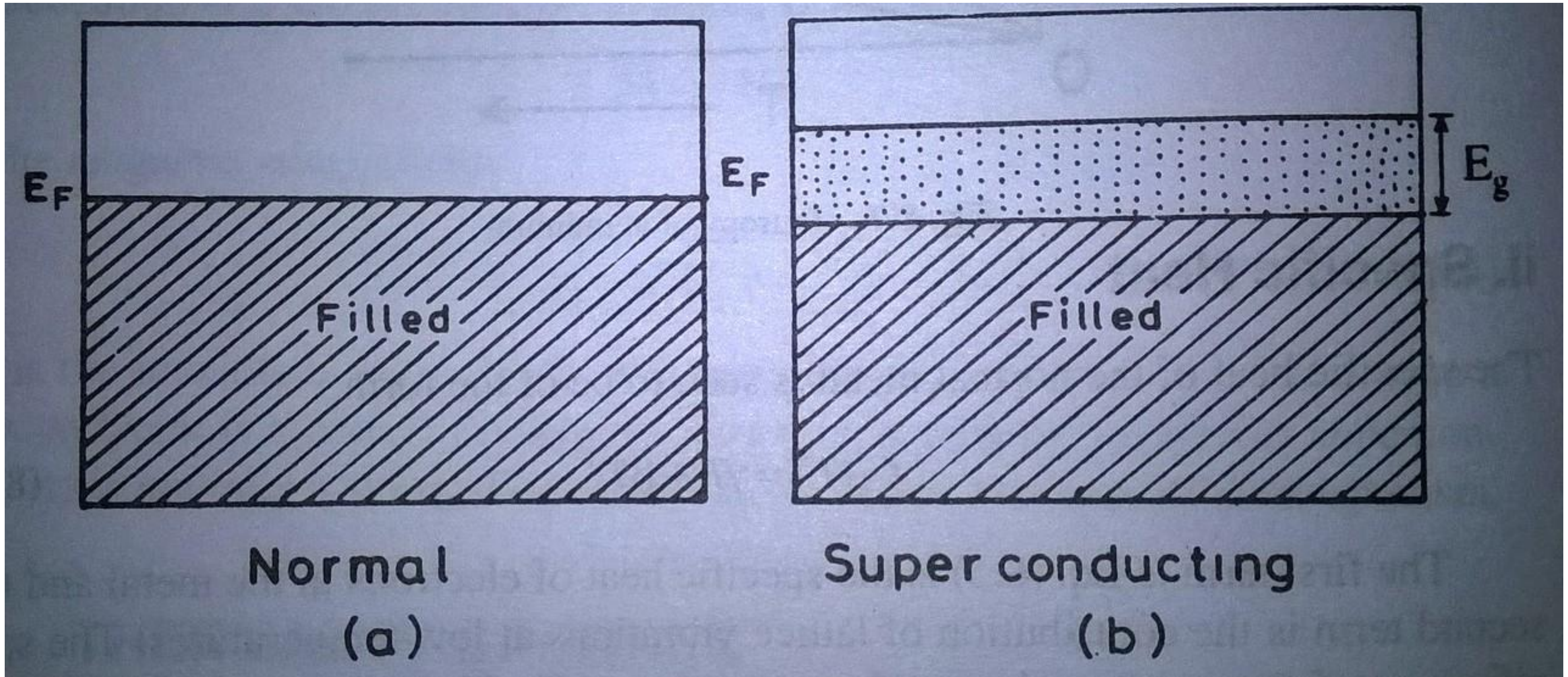


Fig. 8.9 (a) Conduction band in the normal state.
(b) Energy gap at the fermi level in the superconducting state $E_g = 10^{-4}$ eV.

Energy Gap

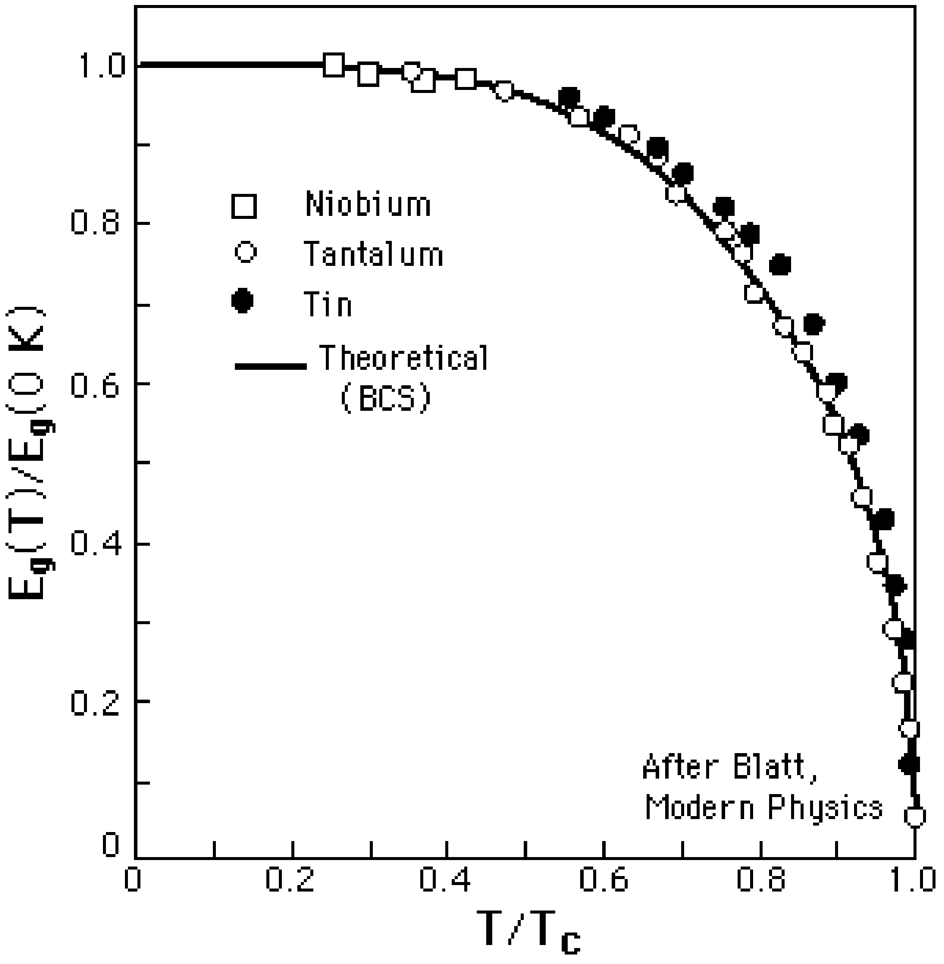
- The energy gap of superconductors is of entirely different nature than the energy gap in insulators
- In superconductor the energy gap is due to electron-electron interaction in Fermi gas whereas in insulator or semiconductor the energy gap is caused by electron lattice interaction
- In insulators the gap prevents the flow of electrical current. Energy must be added to lift electrons from the valence band to conduction band before the current can flow
- In a superconductor, on the other hand, the current flows despite the presence of energy gap
- In a superconductor the electrons in the excited state above the gap behave as normal electron
- The electrons in a superconducting material behave as normal electrons above temperature T_c, but the electrons are paired (Cooper pair) at temperature below T_c with total energy less than E_F

Energy Gap

➢The difference between the normal state & paired state electrons appears as energy gap E_g at the Fermi surface

$$E_g = 2b(k_B T_c), \quad 2b \approx 3.5$$

$$T_c = 1.2 \text{ K for Aluminium}$$
$$E_g = 2b(k_B T_c) = 3.5 \times 1.38 \times 10^{-23} \times 1.2 \text{ J}$$
$$= 0.36225 \text{ meV}$$



BCS Theory - Outlines

- Microscopic theory put forward by **Bardeen–Cooper–Schrieffer** (BCS)

John Bardeen Leon Cooper Bob Schrieffer

Nobel Prize 1972 for their theory of 1957 which explained conventional superconductors: nearly 50 years after the discovery by Kamerlingh Onnes!

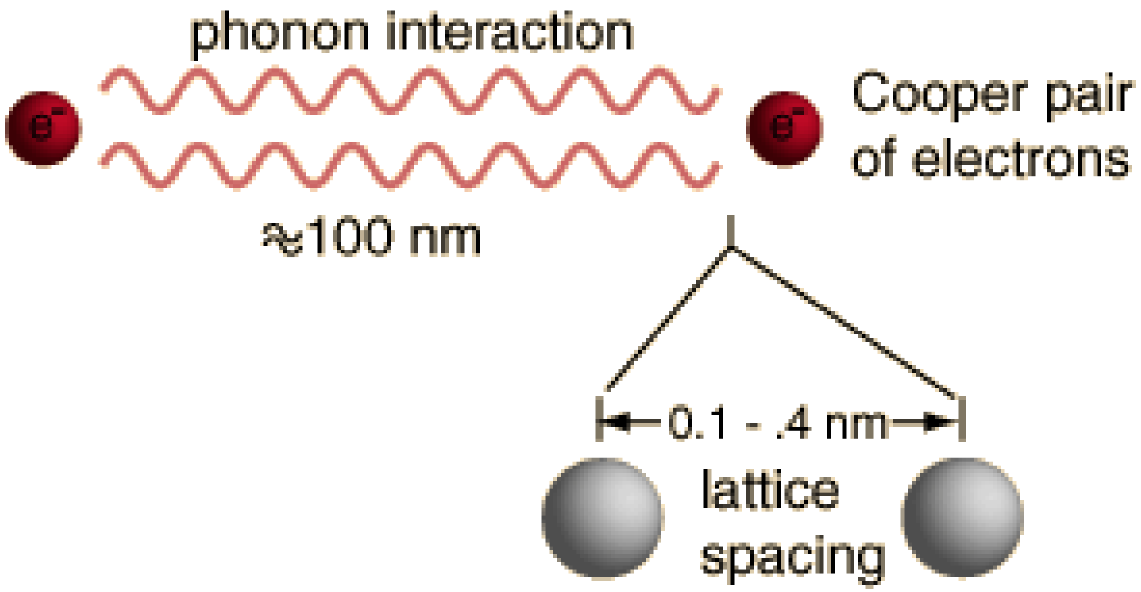
BCS Theory - Outlines

Electron-phonon-electron interaction:

- Fundamental postulate of BCS theory is that when an attractive interaction between two electrons by means of phonon exchange dominates the repulsive coulomb interaction then the superconducting state is formed
- During an interaction of an electron with a positive ion of the lattice through electrostatic coulomb force, some electron momentum get transferred
- As a result, these ions set up elastic wave in the lattice due to distortion
- If another electron happens to pass through this region then the interaction between two occurs which in its effect lowers the energy of the second electron

BCS Theory - Outlines

- The two electrons interact via the lattice distortion or the phonon field resulting in the lowering of energy of the electron which implies the force between two electrons is attractive
- This interaction is strongest when two electrons have equal and opposite moments and spin and this pair is known as **cooper pair**



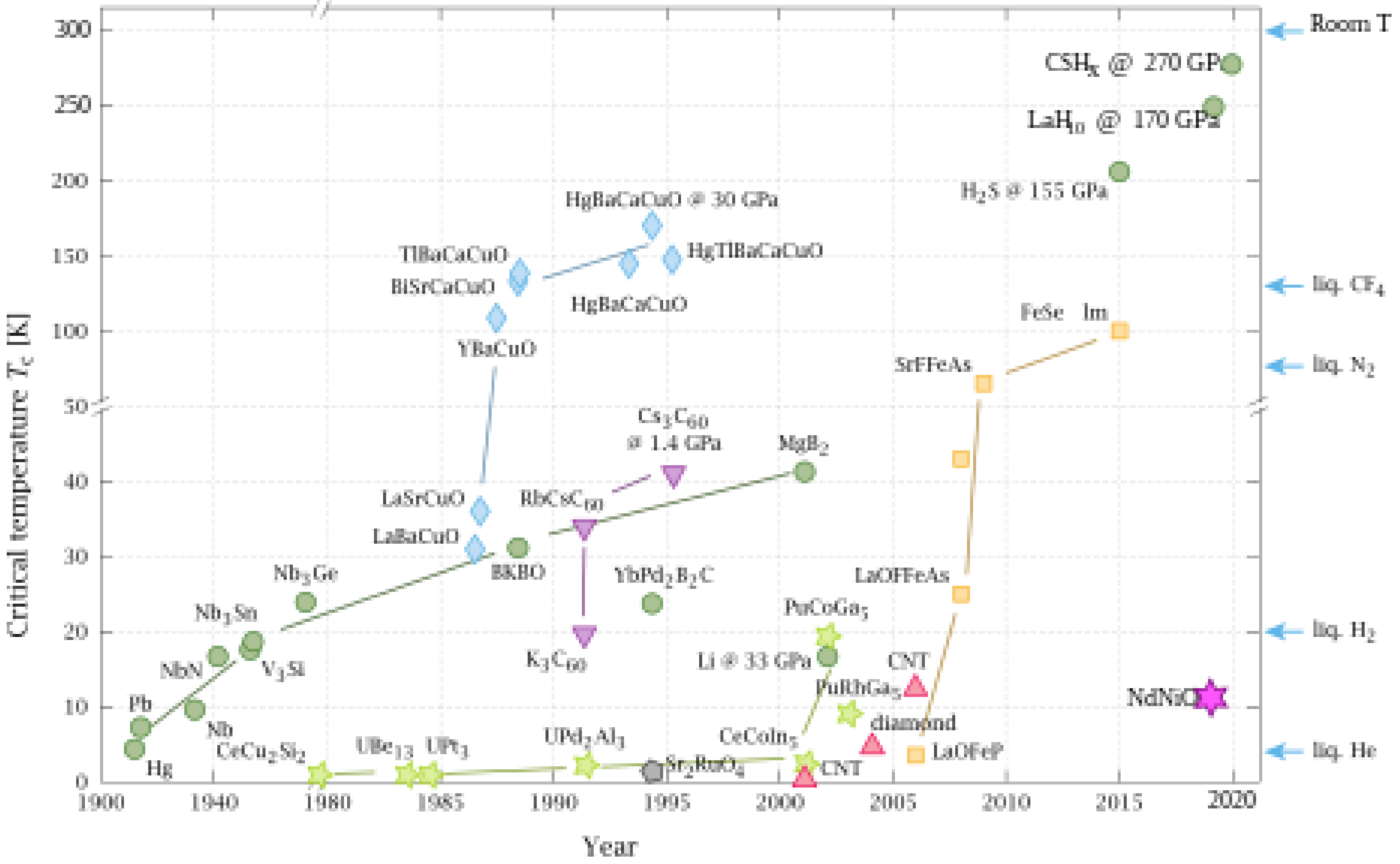
BCS Theory - Outlines

- When the temperature of the specimen is lowered, if the attractive force between two electrons via a phonon exceeds coulomb repulsion between them, then a weakly bound cooper pair is formed having the binding energy of the order of 10^{-3} eV. The energy of Cooper pair is less than the energy of the pair in free state. The binding energy of cooper pair is called energy bang gap, E_g. When $h\nu \geq E_g$ strong absorption occurs as the cooper pairs break apart.
- The electrons in cooper pair have opposite spins so the total spin of the pair is zero. As a result cooper pairs are bosons whereas electrons are fermions.

High T_c Superconductors – Idea

- HTSC – a new class of superconducting materials with high T_c values
- Bear extraordinary superconducting and magnetic properties & have great potential for wide-ranging technological applications
- Ordinary metal superconductors have T_c below 20 K where as HTSC have been observed with T_c as high as 138 K
- HTSC materials are not metal or intermetallic compounds but oxides of copper in combination with other elements
- In 1983, 1987 & 1988 materials with T_c upto 40 K, 93 K, 125 K discovered respectively

High T_c Superconductors – Idea



Applications

- High-field magnet applications
- NMR – Nuclear Magnetic Resonance (medical diagnostics & spectroscopy)
- Magnetic levitation – high speed train
- Magnetic shielding
- High & stable magnetic field production – mass spectrometer (Penning trap), colliders etc.
- Energy storage & electric power transmission
- SQUIDS – Superconducting Quantum Interference Devices
- Josephson devices – square-law detector, parametric amplifier, mixer
- Semiconductor – superconductor hybrids (A-D converter) for computer application
- Superconducting quantum computing
- Optoelectronics applications

Review of CU Exam. Papers

CU – 2019

1. Write down two important differences between normal superconductor and High-Tc superconductor. [2]
2. Sketch the specific heat of a superconductor and a normal metal as a function of temperature in the same graph. What information can be obtained from the above graph? [2+1]
3. Distinguish between type I and type II superconductors with the help of M-H plot. [2]
4. A given superconductor has critical magnetic fields 1.4×10^5 A/m and 4.2×10^5 A/m at 14 K and 13 K respectively. Compute the transition temperature and the critical magnetic field at 0 K. [3]

Review of CU Exam. Papers

CU – 2018

1. What are the differences of energy gap seen in superconductor and semiconductor? [2]
2. Explain ‘isotope effect’ in superconductivity. Briefly discuss its significance. [2+1]
3. Derive the behavior of magnetic field inside the superconductor. Hence, define the characteristic length scale. [2+1]

CU – 2017

1. Sketch the specific heat of a superconductor and normal metal as a function of temperature. (Indicate the critical transition temperature in the graph). [2]
2. Explain briefly the Meissner effect with a suitable diagram. [2+1]
3. Calculate the wavelength of the photon which will be required to destroy the superconductivity in Aluminium having critical transition temperature 1.2 K. In which region of electromagnetic spectrum does it belong? [1+1]

Review of CU Exam. Papers

CU – 2016

1. Is Meissner effect consistent with disappearance of resistivity in a superconductor? Explain. [2]
2. In which ways, the energy gaps seen in superconductor are different from semiconductor? How does the energy gap vary with temperature? Sketch the variation of specific heat of a superconductor with temperature (show also the critical transition temperature T_c in the graph) [2+1+2]

CU – 2015

1. Sketch the specific heat of a superconductor and normal metal as a function of temperature in the same graph. [2]
2. Distinguish between type I and type II superconductors with the help of M – H plot. [2]
3. What is the implication of isotope effect? Given that the transition temperature of ordinary mercury (Hg) of atomic mass 200.59 is 4.153 K, find out the transition temperature of mercury having atomic mass 204. [2+1]

Review of CU Exam. Papers

CU – 2014

1. What is the experimental observation of the presence of an energy gap in a superconductor? [2]
2. Explain briefly the Meissner effect with a suitable diagram. Show that the magnetic field decays inside the superconductor exponentially with a characteristic length scale. [(2+1)+3]

CU – 2013

1. For a superconductor, what do you mean by critical temperature and critical field? [2]
2. Explain the meaning of energy gap in a superconductor. [2]
3. What is isotope effect? What is its importance in the theory of superconductivity? [2+1]