Drude's Theory

April 4, 2024

Syllabus: Free electron gas in metals, effective mass, drift current, mobility and conductivity, Hall effect in metals. Thermal conductivity. Lorentz number, limitation of Drude's theory.

This is a lecture note and it does not include the details of the topics. For an insight, the students are advised to consult text books and other references. While preparing this note, I have exclusively taken the help from many sources. Of particular to mention, are *Solid State Physics* by A. J. Dekker, *Introduction to Solid State Physics* by C. Kittel and *Wikipedia*. I have frequently quoted the statements and explanations which I could not make lucid as they appear in these references.

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1 Introduction

In Drude's postulate it is assumed that the valence electrons in metals are surrounded by positive ion cores and effectively do not experience any net force. These electrons, in absence of external field, move in constant potential due to thermal agitation. In many respects, the behaviour of these electrons resemblances to that of ideal gas and it is, therefore, referred to as free electron gas.

2 Wiedmann Franz law

Let us consider a metal having free electron density n. An external electric field of strength F produces an acceleration $a = eF/m$ and hence, the velocity gained by the electrons over a time interval τ is $v = a\tau = \dfrac{eF}{m}\tau$. If $\overline{\lambda}$ be the mean free path of the electrons having some average speed $\overline{c}$, $\tau = \overline{\lambda}/\overline{c}$ and hence the average velocity or the drift velocity gained by the electrons over the relaxation period τ is

$$v_d = \frac{v}{2} = \frac{1}{2}\frac{eF}{m}\tau = \frac{eF\overline{\lambda}}{2m\bar{c}} \tag{1}$$

The current density due to the drift motion of these free electrons is given by

$$J = nev_d = \frac{ne^2F\overline{\lambda}}{2m\bar{c}} \tag{2}$$

By Ohm's law, $J = \sigma F$ where σ is the conductivity of the medium. Thus, according to the Lorentz-Drude model, the conductivity is given by

$$\sigma = \frac{ne^2\overline{\lambda}}{2m\bar{c}} \tag{3}$$

As the electron gas is treated as an ideal gas, the average energy of the electrons at an equilibrium temperature T is $E = 3k_BT/2$. Following the kinetic theory of gases, the thermal conductivity of the electron gas is

$$K = \frac{1}{3}n\bar{c}\bar{\lambda}\frac{dE}{dT} = \frac{1}{2}n\bar{c}\bar{\lambda}k_B \tag{4}$$

Thus

$$\begin{aligned} \frac{K}{\sigma} = \frac{mk_B}{e^2}\bar{c}^2 &= \frac{8}{\pi}\left(\frac{k_B}{e}\right)^2 T \\ \Rightarrow \frac{K}{\sigma T} &= \frac{8}{\pi}\left(\frac{k_B}{e}\right)^2 = L \end{aligned} \tag{5}$$

where $L = \frac{8}{\pi}\left(\frac{k_B}{e}\right)^2$ is constant and known as the Lorentz number. We have used the relation for the average speed of electron gas $\bar{c} = \sqrt{8k_BT/\pi m}$. Equation 5 is known as Wiedmann-Franz law.

Wiedmann-Franz law can alternatively be derived quantum mechanically following the Sommerfeld model (free electron Fermi gas). The thermal conductivity, mean free path and average speed of the electrons are slightly modified from what we have derived following the classical approach due to Drude and the Lorentz number is given by $L = \frac{1}{3}\left(\frac{\pi k_B}{e}\right)^2 = 2.45 \times 10^{-8}$ W-Ω/kelvin^2 which agrees well with the experimental values.

The resistance of a uniform copper wire of length 0.5m and diameter 0.3 mm is 0.13 ω at room temperature. The thermal conductivity of copper is 390 W-$m^{-1}K^{-1}$. Determine the Lorentz number and compare it with the theoretical value.

Length $l = 0.5$ m, radius $r = 0.15$ mm $= 1.5 \times 10^{-4}$ m. The resistance $R = 0.13\ \Omega$. If σ be the electrical conductivity of copper at the room temperature ($T = 300$ K),

$$\sigma = \frac{l}{AR} = \frac{l}{\pi r^2 R} = \frac{0.5}{3.14 \times (1.5)^2 \times 10^{-8} \times 0.13} = 5.4 \times 10^7 \text{ S/m}$$

Lorentz number from this data is

$$L = \frac{K}{\sigma T} = \frac{390}{5.4 \times 10^7 \times 293} = 2.45 \times 10^{-8} \text{ W-}\Omega/\text{kelvin}^2$$

Theoretically, Lorentz number is given by

$$L = \frac{1}{3}\left(\frac{\pi k_B}{e}\right)^2 = 2.45 \times 10^{-8} \text{ W-}\Omega/\text{kelvin}^2$$

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3 Hall Effect

The Hall effect, named after its discoverer Edwin Hall (1879), is the production of a voltage difference (termed as the Hall voltage V_H) across a conductor or semiconductor, transverse to an electric current (J_x) in the medium and to an applied magnetic field (B_z) perpendicular to the current. The Hall coefficient is defined as the ratio of the induced electric field to the product of the current density and the applied magnetic field:

$$R_H = \frac{E_H}{J_x B_z} \tag{6}$$

The Hall coefficient R_H is the characteristic of the material from which the conductor/semiconductor is made, since its value depends on the type, number, and properties of the charge carriers that constitute the current.

3.1 Hall effect in metals

Consider a metal chip carrying some current along the +ve x-direction and placed in an external field $\vec{B} = B\hat{z}$. The applied electric field driving the current is $\vec{E} = E\hat{x}$ and the drift velocity of the electrons (charge $q = -e$) is $\vec{v} = -v\hat{x}$. The current density is given by $\vec{J} = nq\vec{v} = nev\hat{x}$. The electrons, on their drift motion, experience a Lorentz force $\vec{F}_L = q(\vec{v} \times \vec{B}) = evB\hat{y}$. Thus the electrons are deflected along the y-direction due to the Lorentz force and a concentration gradient is developed between the opposite surfaces along this direction. This, in turn, produces an electric field which prevents further

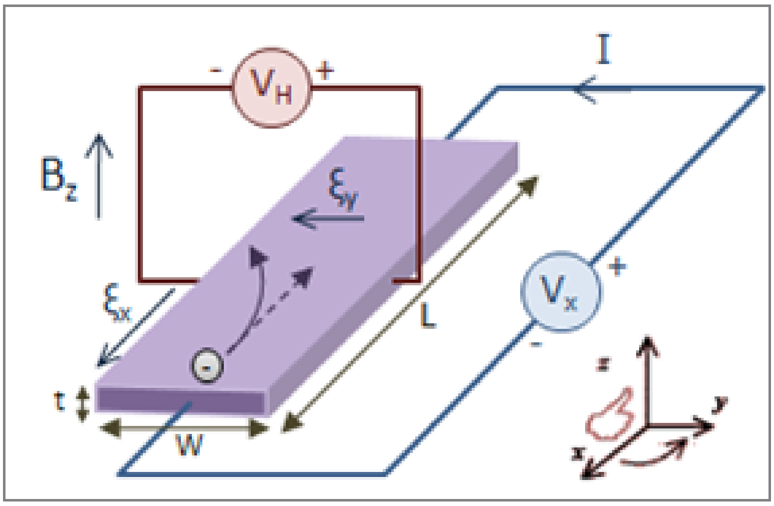

Figure 1: Hall effect measurement setup in metals (Wikipedia)

deflection of electrons along the y-direction. The Lorentz force is balanced by the force produced due to the Hall effect. Denoting the Hall field by $\vec{E}_H$, we have

$$\begin{aligned} -e\vec{E}_H &= -\vec{F}_L = evB\hat{y} \\ \implies \vec{E}_H &= -vB\hat{y} \end{aligned} \tag{7}$$

The Hall coefficient is defined as

$$R_H = \frac{E_H}{JB} = \frac{-vB}{nevB} = -\frac{1}{ne} \tag{8}$$

The Hall coefficient could be related with the electron mobility μ and the electrical conductivity σ of the material. Note that, $v = \mu E$ and $E_H = vB = \mu EB$. Thus, the Hall coefficient $R_H = E_H/JB = \mu/\sigma$ since $J = \sigma E$.