The Stackelberg Model

In oligopoly market structure firms are interdependent. Therefore, the oligopolistic or duopolistic firms are to take decisions on the basis of assumptions about the reaction patterns of its rivals to its own decision. The reaction pattern can be expressed in the form of an equation - called reaction function.

In Cournot model we assume that the oligopolists make their output decisions at the same time. But in Stackelberg model one firm sets its output first before the other firms and so the firm that sets its output first gets the first mover advantage. In fact the firm is sophisticated enough to discover the reaction function of its rival in a duopoly, consequently it would be able to maximize its profit function with respect to its own output level. Such a sophisticated firm is called a leader. On the other hand if the rival firm cannot discover the leader's reaction function, but obeys its own reaction function then the rival firm would be called a follower. The duopoly model based on such conjectural behaviour of firms has been developed by the German economist Stackelberg. The model can be summarized as follows:

Generally the profit of each duopolist is a function of the output levels of both,

$$\pi_1 = h_1(Q_1, Q_2) \qquad\qquad (1)$$

$$\pi_2 = h_2(Q_1, Q_{2)}) \qquad\qquad (2)$$

where π_i and Q_i (i = 1,2) are the ith firm's profit and output .

The reaction functions of the firms can be written as,

$$Q_1 = f_1(Q_2) \qquad\qquad\qquad (3)$$

$$Q_2 = f_2(Q_1) \qquad\qquad\qquad (4)$$

If firm I is sophisticated enough to discover firm II's reaction function which is given in equation (4), then this firm would like to act as a leader. In that case firm I will substitute firm II's reaction function into its profit function:

$$\pi_1 = h_1\{Q_1, f_2(Q_1)\} = \phi_1(Q_1) \qquad\qquad (5)$$

Equation (5) shows that π_1 is a function of Q_1 alone and hence it can be maximized with respect to Q_1. If firm II acts as a follower and obeys its reaction function, then a stable equilibrium will be obtained by substituting I's profit maximising output into the former's reaction function.

But if both the firms are sophisticated enough to discover each other's reaction functions and like to act as leaders, then there will be a Stackelberg disequilibrium.

The model can be explained dramatically as follows:
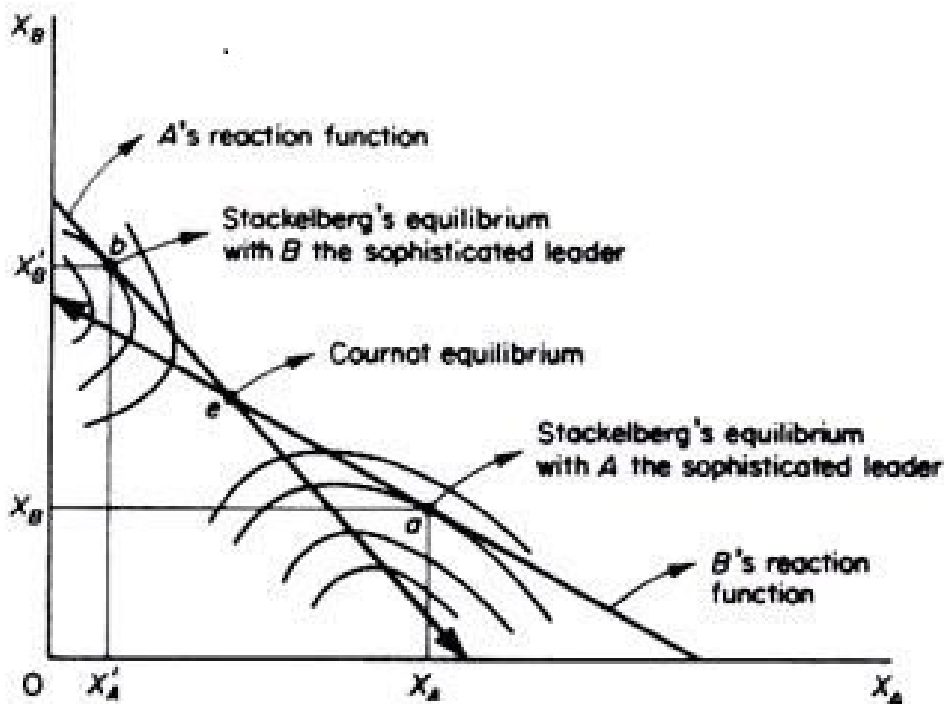
In the above diagram the curves concave to the horizontal axis (X_A axis) are firm I's isoprofit curves and those concave to the vertical axis (X_B axis) are firm II's isoprofit curves. Here we see that if firm I knows firm II's reaction function, then the former would like to act as the leader and hence it would decide to produce X_A amount of output in order to earn the highest possible profit. Since the isoprofit curves are concave to the firm's output axis and the isoprofit curve lying closer to that axis represents a higher profit, the point of tangency between the firm's reaction curve and the leader 's isoprofit curve would give the latter's equilibrium output. So if firm I acts as a leader then it would decide to produce X_A amount of output; if firm II acts as a leader then it would like to produce X_B amount of output.

Let us consider an example. Suppose the market demand curve is given by P = 30 - Q, where Q is the total output. We also assume that both firms have zero marginal cost. Since firm II makes its output decision after firm I, it takes firm I's output as fixed. Firm II's reaction function is given by,

$$MR_2 = 0$$
$$Q_2 = 15 - 1/2Q_1 \qquad\qquad (6)$$

Firm I's revenue is given by,

$$R_1 = PQ_1 = 30Q_1 - Q_1^2 - Q_2 Q_1 \qquad\qquad (7)$$

Firm I knows firm II's reaction function, i.e. firm I knows how much firm II will produce for a particular value of Q_1. So, putting the expression for Q_2 from equation (6) in equation (7), we find,

$$R_1 = 15 Q_1 - 1/2Q_1^2$$

Its marginal revenue is therefore,

$MR_1 = 15 - Q_1$

Setting $MR_1 = 0$ (as given $MC = 0$) gives $Q_1 = 15$. From firm II's reaction curve, we find that $Q_2 = 7.5$. It is clear that firm I produces twice as much as firm II and makes twice as much profit. So firm I gets first mover's advantage.