INTERPOLATION WITH UNEQUALLY SPACED POINTS

(Numerical analysis practical)

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Problem statement:-

Compute the values of $f(x)$ for $x = 0.42$ and $x = 0.52$ from the following table by using **Lagrange's interpolation formula** :

x	$y = f(x)$
0.30	3.106234
0.34	3.109779
0.40	3.115119
0.48	3.122278
0.53	3.126777
0.60	3.133104
0.65	3.137646

Working formula:-

Lagrange's interpolation formula (without error term) is given by

$$y = f(x) \approx \sum_{r=0}^{n} \frac{\omega(x)}{(x - x_r)\omega'(x_r)} f(x_r) = \omega(x) \sum_{r=0}^{n} \frac{y_r}{D_r},$$

where

$$\omega(x) = (x - x_0)(x - x_1)\dots(x - x_{r-1})(x - x_r)(x - x_{r+1})\dots(x - x_n),$$

$$\omega'(x_r) = (x_r - x_0)(x_r - x_1)\dots(x_r - x_{r-1})(x_r - x_{r+1})\dots(x_r - x_n),$$

$$D_r = (x - x_r)(x_r - x_0)(x_r - x_1)\dots(x_r - x_{r-1})(x_r - x_{r+1})\dots(x_r - x_n),\ r = 0, 1, \dots, n.$$

Results:-

$f(0.42) \approx 3.116904$
$f(0.52) \approx 3.125876$

Computation table for Lagrange's interpolation :

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	D_r	y_r	$y_r/D_r(+ve)$	$y_r/D_r(-ve)$
x_0	$x-x_0$	x_0-x_1	x_0-x_2	x_0-x_3	x_0-x_4	x_0-x_5	x_0-x_6	D_0	y_0	y_0/D_0	
x_1	x_1-x_0	$x-x_1$	x_1-x_2	x_1-x_3	x_1-x_4	x_1-x_5	x_1-x_6	D_1	y_1		y_1/D_1
x_2	x_2-x_0	x_2-x_1	$x-x_2$	x_2-x_3	x_2-x_4	x_2-x_5	x_2-x_6	D_2	y_2		y_2/D_2
x_3	x_3-x_0	x_3-x_1	x_3-x_2	$x-x_3$	x_3-x_4	x_3-x_5	x_3-x_6	D_3	y_3	y_3/D_3	
x_4	x_4-x_0	x_4-x_1	x_4-x_2	x_4-x_3	$x-x_4$	x_4-x_5	x_4-x_6	D_4	y_4	y_4/D_4	
x_5	x_5-x_0	x_5-x_1	x_5-x_2	x_5-x_3	x_5-x_4	$x-x_5$	x_5-x_6	D_5	y_5		y_5/D_5
x_6	x_6-x_0	x_6-x_1	x_6-x_2	x_6-x_3	x_6-x_4	x_6-x_5	$x-x_6$	D_6	y_6	y_6/D_6	
Total :										$\sum y_r/D_r$	$\sum y_r/D_r$

Computation table for Lagrange's interpolation :

Setting the transformation $x'_i = (x_i - 0.48) \times 100,\ i = 0(1)6$, then all nodal points transformed to $-18,\ -14,\ -8,\ 0,\ 5,\ 12,\ 17$. To compute $f(0.42)$, we take $x = 0.42$, then $x' = (0.42 - 0.48) \times 100 = -6$.

							D_r	y_r	$\frac{y_r}{D_r}$ (+ve)	$\frac{y_r}{D_r}$ (-ve)
[12]	−4	−10	−18	−23	−30	−35	208856000	3.106234	$1.488686642 \times 10^{-8}$	
4	[8]	−6	−14	−19	−26	−31	−141164032	3.109779		$7.554602523 \times 10^{-8}$
10	6	[2]	−8	−13	−20	−25	6240000	3.115119	$4.992177885 \times 10^{-7}$	
18	14	8	[−6]	−5	−12	−17	12337920	3.122278	$2.530635634 \times 10^{-7}$	
23	19	13	5	[−11]	−7	−12	− 26246220	3.126777		$1.191324694 \times 10^{-7}$
30	26	20	12	7	[−18]	−5	117936000	3.133104	$2.656613757 \times 10^{-8}$	
35	31	25	17	12	5	[−23]	−636352500	3.137646		$4.930672858 \times 10^{-9}$
Total :									$7.937343559 \times 10^{-7}$	$1.996091675 \times 10^{-7}$

$$\therefore \quad \sum_{r=0}^{6} \frac{y_r}{D_r} = 5.941251884 \times 10^{-7} \ \ \& \ \ \omega(-6) = 5246208 \ .$$

$$\therefore \quad f(0.42) = L(-6) = 5246208 \times 5.941251884 \times 10^{-7} = 3.116904316 \approx 3.116904 \ .$$

Computation table for Lagrange's interpolation :

Setting the transformation $x'_i = (x_i - 0.48) \times 100,\ i = 0(1)6$, then all nodal points transformed to $-18,\ -14,\ -8,\ 0,\ 5,\ 12,\ 17$. To compute $f(0.52)$, we take $x = 0.52$, then $x' = (0.52 - 0.48) \times 100 = 4$.

							D_r	y_r	$\frac{y_r}{D_r}$ (+ve)	$\frac{y_r}{D_r}$ (-ve)
22	−4	−10	−18	−23	−30	−35	382536000	3.106234	$8.120108957 \times 10^{-9}$	
4	18	−6	−14	−19	−26	−31	−92619072	3.109779		$3.357601121 \times 10^{-8}$
10	6	12	−8	−13	−20	−25	37490000	3.115119	$8.320296474 \times 10^{-8}$	
18	14	8	4	−5	−12	−17	− 8225280	3.122278		$3.795953451 \times 10^{-7}$
23	19	13	5	−1	−7	−12	− 2386020	3.126777		$1.310457163 \times 10^{-6}$
30	26	20	12	7	−8	−5	52416000	3.133104	$5.977380952 \times 10^{-8}$	
35	31	25	17	12	5	−13	−359677500	3.137646		$8.723498134 \times 10^{-9}$
Total :									$15.10968832 \times 10^{-8}$	$173.235201744 \times 10^{-8}$

$$\therefore \quad \sum_{r=0}^{6} \frac{y_r}{D_r} = -158.125513424 \times 10^{-8} \ \ \& \ \ \omega(4) = -1976832 \ .$$

$$\therefore \quad f(0.52) = L(4) = (-1976832) \times \left(-158.125513424 \times 10^{-8}\right) = 3.125875749 \approx 3.125876 \ .$$

LAGRANGE'S INTERPOLATION PROBLEMS & ANSWERS

Prob.-1: $x = 0.42$, $x = 0.52$.

x	$f(x)$
0.30	$R + 1.106234$
0.34	$R + 1.109779$
0.40	$R + 1.115119$
0.48	$R + 1.122278$
0.53	$R + 1.126777$
0.60	$R + 1.133104$
0.65	$R + 1.137646$

where $R = 0(1)9$.

Ans.: $f(0.42) \approx R + 1.11690432$ & $f(0.52) \approx R + 1.12587575$

Prob.-2: $x = 0.47$, $x = 0.57$.

x	$h(x)$
0.35	$R + 0.075027$
0.39	$R + 0.082723$
0.45	$R + 0.094371$
0.53	$R + 0.110099$
0.58	$R + 0.120044$
0.65	$R + 0.134119$
0.70	$R + 0.144282$

where $R = 0(1)9$.

Ans.: $h(0.47) \approx R + 0.09828186$ & $h(0.57) \approx R + 0.11804784$

Prob.-3: $x = 0.82$, $x = 0.92$.

x	$f(x)$
0.70	$R + 0.242344$
0.74	$R + 0.257845$
0.80	$R + 0.281460$
0.88	$R + 0.313637$
0.93	$R + 0.334157$
1.00	$R + 0.363425$
1.05	$R + 0.384723$

where $R = 0(1)9$.

Ans.: $f(0.82) \approx R + 0.28942961$ & $f(0.92) \approx R + 0.33002748$

Prob.-4: $x = 0.21 + \frac{R+1}{100}$, $x = 0.60 + \frac{R}{100}$, where $R = 0(1)9$.

x	$g(x)$	x	$g(x)$
0.00	1.397500	0.40	1.437750
0.14	1.411460	0.51	1.449031
0.21	1.418493	0.59	1.457285
0.32	1.429615	0.71	1.469754

Ans.:

R	x	$g(x)$	x	$g(x)$
0	0.22	1.4195012809	0.60	1.4583176631
1	0.23	1.4205102307	0.61	1.4593505128
2	0.24	1.4215197910	0.62	1.4603836541
3	0.25	1.4225299141	0.63	1.4614172554
4	0.26	1.4235405638	0.64	1.4624515598
5	0.27	1.4245517160	0.65	1.4634868981
6	0.28	1.4255633590	0.66	1.4645237031
7	0.29	1.4265754927	0.67	1.4655625243
8	0.30	1.4275881283	0.68	1.4666040444
9	0.31	1.4286012872	0.69	1.4676490968

Prob.-5: $x = 10.5 + \frac{R}{100}$, $x = 11.4 - \frac{R}{100}$, where $R = 0(1)9$.

x	$h(x)$
10.5	$R + 0.36969$
10.6	$R + 0.43839$
10.8	$R + 0.49544$
10.9	$R + 0.50022$
11.1	$R + 0.48332$
11.4	$R + 0.42257$

Ans.:

R	x	$h(x)$	x	$h(x)$
0	10.50	0.3696900000	11.40	0.4225700000
1	10.51	1.3782802947	11.39	1.4246814905
2	10.52	2.3864485308	11.38	2.4268256927
3	10.53	3.3942104747	11.37	3.4289969898
4	10.54	4.4015813950	11.36	4.4311900292
5	10.55	5.4085760716	11.35	5.4333997135
6	10.56	6.4152088045	11.34	6.4356211923
7	10.57	7.4214934227	11.33	7.4378498526
8	10.58	8.4274432931	11.32	8.4400813103
9	10.59	9.4330713292	11.31	9.4423114014

Problem statement:-

Compute the values of $g(x)$ for $x = 5.02$ and $x = 5.95$ from the following table by using **Newton's divided difference formula** :

x	$y = g(x)$
5.00	0.3765
5.10	0.3828
5.30	0.3956
5.70	0.4226
5.80	0.4296
6.00	0.4441

Working formula:-

Newton's divided difference formula with x_0 as starting point is given by

$$g(x) \approx g(x_0) + (x - x_0)g[x_0, x_1] + (x - x_0)(x - x_1)g[x_0, x_1, x_2] + \dots$$

$$+(x - x_0)(x - x_1)(x - x_2)\dots(x - x_{n-1})g[x_0, x_1, x_2, \dots, x_n]$$

$$= g_0 + (x - x_0)\chi g + (x - x_0)(x - x_1)\chi^2 g + \dots + (x - x_0)(x - x_1)(x - x_2)\dots(x - x_{n-1})\chi^n g,$$

where

$$\chi g = g[x_0, x_1] = \frac{g(x_0) - g(x_1)}{x_0 - x_1},$$

$$\chi^2 g = g[x_0, x_1, x_2] = \frac{g[x_0, x_1] - g[x_1, x_2]}{x_0 - x_2},$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\chi^n g = g[x_0, x_1, x_2, \dots, x_n] = \frac{g[x_0, x_1, \dots, x_{n-1}] - g[x_1, x_2, \dots, x_n]}{x_0 - x_n}.$$

Results:-

$g(5.02) \approx 0.3778$
$g(5.95) \approx 0.4404$

Computation table for Newton's divided difference interpolation :

				x	$g(x)$	χg	$\chi^2 g$	$\chi^3 g$	$\chi^4 g$
				x_0	$g(x_0)$				
			$x_0 - x_1$			$g[x_0, x_1]$			
		$x_0 - x_2$		x_1	$g(x_1)$		$g[x_0, x_1, x_2]$		
	$x_0 - x_3$		$x_1 - x_2$			$g[x_1, x_2]$		$g[x_0, x_1, x_2, x_3]$	
$x_0 - x_4$		$x_1 - x_3$		x_2	$g(x_2)$		$g[x_1, x_2, x_3]$		$g[x_0, x_1, x_2, x_3, x_4]$
	$x_1 - x_4$		$x_2 - x_3$			$g[x_2, x_3]$		$g[x_1, x_2, x_3, x_4]$	
		$x_2 - x_4$		x_3	$g(x_3)$		$g[x_2, x_3, x_4]$		
			$x_3 - x_4$			$g[x_3, x_4]$			
				x_4	$g(x_4)$				

Computation table for Newton's divided difference :

					x	$g(x)$	χg	$\chi^2 g$	$\chi^3 g$	$\chi^4 g$	$\chi^5 g$
					5.00	0.3765					
				$-.1$			0.063				
			$-.3$		5.10	0.3828		0.0033333			
		$-.7$		$-.2$			0.064		.0035714		
	$-.8$		$-.6$		5.30	0.3956		0.0058333		$-.0059524$	
$-.1$		$-.7$		$-.4$			0.0675		$-.0011905$		.0125661
	$-.9$		$-.5$		5.70	0.4226		0.005		.0066138	
		$-.7$		$-.1$			0.0700		.0047619		
			$-.3$		5.80	0.4296		0.0083333			
				$-.2$			0.0725				
					6.00	0.4441					

Here $x = 5.02, \quad x_0 = 5.00$.

Coefficient	Multiplier	Positive term	Negative term
1.0	0.3765	0.3765	
0.2	0.063	0.00126	
−0.0016	0.0033333		0.0000053
0.000448	0.0035714	0.0000016	
−0.00030464	−0.0059524	0.0000018	
0.00023762	0.0125661	0.000003	
Total :-		0.3777664	0.0000053

$$\begin{array}{r} 0.3777664 \\ -0.0000053 \\ \hline 0.3777611 \end{array}$$

$$\therefore \quad g(5.02) \approx 0.3778 \ .$$

Here $x = 5.95, \quad x_5 = 6.00$.

Coefficient	Multiplier	Positive term	Negative term
1.0	0.4441	0.4441	
−0.05	0.0725		0.003625
−0.0075	0.0083333		0.0000625
−0.001875	0.0047619		0.0000089286
−0.00121875	0.0066138		0.0000080605
−0.0010359375	0.0125661		0.0000130177
Total :-		0.4441	0.0037175068

$$\begin{array}{r} 0.4441000000 \\ -0.0037175068 \\ \hline 0.4403824932 \end{array}$$

$$\therefore \quad g(5.95) \approx 0.4404 \ .$$

Note : Here we use the Newton's divided difference formula with x_5 as starting point :

$$g(x) \approx g(x_5) + (x - x_5)\ g[x_5, x_4] + (x - x_5)(x - x_4)\ g[x_5, x_4, x_3] + (x - x_5)(x - x_4)$$
$$(x - x_3)\ g[x_5, x_4, x_3, x_2] + (x - x_5)(x - x_4)(x - x_3)(x - x_2)\ g[x_5, x_4, x_3, x_2, x_1]$$
$$+(x - x_5)(x - x_4)(x - x_3)(x - x_2)(x - x_1)\ g[x_5, x_4, x_3, x_2, x_1, x_0] \ .$$

NEWTON'S DIVIDED DIFFERENCE INTERPOLATION PROBLEMS & ANSWERS

Prob.-1: $x = 5.00 + \frac{R+1}{100}$, $x = 6.00 - \frac{R+1}{100}$, where $R = 0(1)9$.

x	$f(x)$	x	$f(x)$
5.00	0.3765	5.70	0.4226
5.10	0.3828	5.80	0.4296
5.30	0.3956	6.00	0.4441

Ans.:

R	x	$f(x)$	x	$f(x)$
0	5.01	0.37713079	5.99	0.44334978
1	5.02	0.37776107	5.98	0.44260309
2	5.03	0.37839096	5.97	0.44185975
3	5.04	0.37902061	5.96	0.44111961
4	5.05	0.37965015	5.95	0.44038249
5	5.06	0.38027968	5.94	0.43964825
6	5.07	0.38090933	5.93	0.43891673
7	5.08	0.38153920	5.92	0.43818779
8	5.09	0.38216939	5.91	0.43746130
9	5.10	0.38280000	5.90	0.43673714

Prob.-2: $x = 0.43$, $x = 0.55$.

x	$f(x)$
0.30	$R + 1.106234$
0.34	$R + 1.109779$
0.40	$R + 1.115119$
0.48	$R + 1.122278$
0.53	$R + 1.126777$
0.60	$R + 1.133104$
0.65	$R + 1.137646$

where $R = 0(1)9$.

Ans.: $f(0.43) \approx R + 1.11779804$ & $f(0.55) \approx R + 1.12858152$

Prob.-3: $x = 0.40 + \frac{R+1}{100}$, $x = 1.50 + \frac{R+1}{100}$, where $R = 0(1)9$.

x	$f(x)$	x	$f(x)$
0.10	1.297424	1.30	1.974630
0.40	1.441063	1.50	2.117807
0.60	1.545552	1.90	2.436058
1.00	1.777808	2.00	2.522829

Ans.:

R	x	$f(x)$	x	$f(x)$
0	0.41	1.4461155616	1.51	2.1252323266
1	0.42	1.4511858384	1.52	2.1326836877
2	0.43	1.4562738927	1.53	2.1401611747
3	0.44	1.4613797867	1.54	2.1476648791
4	0.45	1.4665035831	1.55	2.1551948927
5	0.46	1.4716453444	1.56	2.1627513077
6	0.47	1.4768051338	1.57	2.1703342166
7	0.48	1.4819830144	1.58	2.1779437122
8	0.49	1.4871790497	1.59	2.1855798877
9	0.50	1.4923933032	1.60	2.1932428365

Prob.-4: $x = 0.86$, $x = 0.96$.

x	$g(x)$
0.75	$R + 0.526037$
0.79	$R + 0.534307$
0.85	$R + 0.546825$
0.93	$R + 0.563727$
0.98	$R + 0.574415$
1.05	$R + 0.589541$
1.10	$R + 0.600462$

where $R = 0(1)9$

Ans.:

$$g(0.86) \approx R + 0.5489245 \quad \& \quad g(0.96) \approx R + 0.57012823$$

Prob.-5: $x = 0.28$, $x = 0.63$.

x	$h(x)$
0.00	$R + 0.397500$
0.14	$R + 0.411460$
0.21	$R + 0.418493$
0.32	$R + 0.429615$
0.40	$R + 0.437750$
0.51	$R + 0.449031$
0.59	$R + 0.457285$
0.71	$R + 0.469754$

where $R = 0(1)9$

Ans.: $h(0.28) \approx R + 0.42556336$ & $h(0.63) \approx R + 0.46141726$

Problem statement:-

Compute the positive real root of the following equation lying in (a, b), where $b - a = 0.04$ correct to **4D** by using **Lagrange's inverse interpolation formula** : $f(x) = x^3 - 5.3x^2 + 22.54x - 30.63 = 0$.

Working formula:-

To find the root, we take $y = 0$ and this is a problem of inverse interpolation.
We first find the values of $f(x) = x^3 - 5.3x^2 + 22.54x - 30.63$.
$\therefore$ $f(1.89) = -0.210261 < 0$, $f(1.90) = -0.078 < 0$, $f(1.91) = 0.054341 > 0$, $f(1.92) = 0.186768 > 0$, $f(1.93) = 0.319287 > 0$. Thus the root lies between 1.89 and 1.93.
Now $f'(x) = 3x^2 - 10.6x + 22.54 > 0$ for the interval $(1.89, 1.93)$, so in this interval unique root exist.
We find the root by using **Lagrange's inverse interpolation formula**.
The table is

y	$x = f^{-1}(y) = F(y)$
-0.210261	1.89
-0.078000	1.90
0.054341	1.91
0.186768	1.92
0.319287	1.93

Lagrange's inverse interpolation formula is given by

$$x = F(y) \approx \sum_{r=0}^{n} \frac{\omega(y)}{(y - y_r)\omega'(y_r)} F(y_r) = \omega(y) \sum_{r=0}^{n} \frac{x_r}{D_r},$$

where

$$\omega(y) = (y - y_0)(y - y_1) \dots (y - y_{r-1})(y - y_r)(y - y_{r+1}) \dots (y - y_n),$$

$$\omega'(y_r) = (y_r - y_0)(y_r - y_1) \dots (y_r - y_{r-1})(y_r - y_{r+1}) \dots (y_r - y_n),$$

$$D_r = (y - y_r)(y_r - y_0)(y_r - y_1) \dots (y_r - y_{r-1})(y_r - y_{r+1}) \dots (y_r - y_n), \ r = 0(1)n.$$

Result:-

The real root of the equation $f(x) = 0$ correct up to four decimal places is $x \approx 1.9059$.

Computation table for Lagrange's inverse interpolation :

	y_0	y_1	y_2	y_3	y_4	D_r	x_r	$x_r/D_r(+)$	$x_r/D_r(-)$
y_0	$\boxed{y - y_0}$	$y_0 - y_1$	$y_0 - y_2$	$y_0 - y_3$	$y_0 - y_4$	D_0	x_0	x_0/D_0	
y_1	$y_1 - y_0$	$\boxed{y - y_1}$	$y_1 - y_2$	$y_1 - y_3$	$y_1 - y_4$	D_1	x_1		x_1/D_1
y_2	$y_2 - y_0$	$y_2 - y_1$	$\boxed{y - y_2}$	$y_2 - y_3$	$y_2 - y_4$	D_2	x_2		x_2/D_2
y_3	$y_3 - y_0$	$y_3 - y_1$	$y_3 - y_2$	$\boxed{y - y_3}$	$y_3 - y_4$	D_3	x_3	x_3/D_3	
y_4	$y_4 - y_0$	$y_4 - y_1$	$y_4 - y_2$	$y_4 - y_3$	$\boxed{y - y_4}$	D_4	x_4	x_4/D_4	x_4/D_4
Total :								$\sum x_r/D_r$	$\sum x_r/D_r$

Computation table for Lagrange's inverse interpolation :

Setting the transformation $y'_i = y_i \times 1000000$, $i = 0(1)4$, then all nodal points transformed to -210261, -78000, 54341, 186768, 319287. To find the root, we take $y = 0$, then $y' = y \times 1000000 = 0$.

					D_r $(\times 10^{27})$	x_r	$\frac{x_r}{D_r}$ (+ve) $(\times 10^{-27})$	$\frac{x_r}{D_r}$ (-ve) $(\times 10^{-27})$
210261	−132261	−264602	−397029	−529548	1.547074444	1.89	1.221660669	
132261	78000	−132341	−264768	−397287	−0.143611979	1.90		13.23009416
264602	132341	−54341	−132427	−264946	−0.066765033	1.91		28.60778932
397029	264768	132427	−186768	−132519	0.344544153	1.92	5.572580426	
529548	397287	264946	132519	−319287	−2.358448732	1.93		0.818334515
Total :							6.794241095	42.65621800

$$\therefore \sum_{r=0}^{4} \frac{x_r}{D_r} = -3.58619769 \times 10^{-26} \ \ \& \ \ \omega(0) = -5.3145275 \times 10^{25} .$$

$$\therefore \ x = \left(-5.3145275 \times 10^{25}\right) \times \left(-3.58619769 \times 10^{-26}\right) = 1.905894624 \approx 1.9059 \text{ (correct to 4D).}$$

LAGRANGE'S INVERSE INTERPOLATION PROBLEMS & ANSWERS

Prob.-1:

$$x^3 - px^2 + 22.54x - 30.63 = 0, \; p = 5 + \frac{2R+1}{10}, \quad (1, 3).$$

Ans.:

R	Real root	R	Real root
0	1.853883	5	2.191973
1	1.905895	6	2.294484
2	1.964067	7	2.417376
3	2.029788	8	2.567528
4	2.104913	9	2.754033

Prob.-2:

$$ax^3 - 4x^2 + 8x - 3 = 0, \; a = 1 + \frac{R}{10}, \quad (0, 1).$$

Ans.:

R	Real root	R	Real root
0	0.474043	5	0.463874
1	0.471892	6	0.462000
2	0.469804	7	0.460172
3	0.467773	8	0.458389
4	0.465798	9	0.456648

Prob.-3:

$$x^3 - bx^2 + 8x - 7 = 0, \; b = 3.5 + \frac{R}{20}, \quad (1, 2).$$

Ans.:

R	Real root	R	Real root
0	1.379341	5	1.517788
1	1.403551	6	1.551640
2	1.429331	7	1.588055
3	1.456841	8	1.627284
4	1.486261	9	1.669588

Prob.-4:

$$ax^3 - 18.47x^2 + 12.59x - 19.77 = 0,\ a = 6.6 + \frac{R}{10},\quad (2,3).$$

Ans.:

R	Real root	R	Real root
0	2.513678	5	2.350948
1	2.478870	6	2.321540
2	2.445247	7	2.293070
3	2.412753	8	2.265495
4	2.381337	9	2.238777

Prob.-5:

$$bx^3 - 6.87x^2 - 3.47x + 8.17 = 0,\ b = 1.25 + R,\quad (-2,0).$$

Ans.:

R	Real root	R	Real root
0	−1.241172	5	−0.960909
1	−1.155395	6	−0.929843
2	−1.090758	7	−0.902587
3	−1.039320	8	−0.878377
4	−0.996869	9	−0.856650

Prob.-6:

$$px^3 + 1.6028x^2 + 7.8084x - 16.7664 = 0,\ p = 1 + \frac{R}{10},\quad (1,2).$$

Ans.:

R	Real root	R	Real root
0	1.397200	5	1.330852
1	1.382566	6	1.319345
2	1.368679	7	1.308317
3	1.355470	8	1.297730
4	1.342879	9	1.287554

Prob.-7:

$$bx^3 - 2.3x^2 + ax - 5 = 0,\ b = 2 + \frac{R}{10},\ a = 8 - b,\ \ (0, 1).$$

Ans.:

R	Real root	R	Real root
0	0.900744	5	0.912084
1	0.903215	6	0.914075
2	0.905579	7	0.915984
3	0.907841	8	0.917817
4	0.910008	9	0.919578

Prob.-8:

$$px^3 + 1.52x^2 - 3.21x - 6.84 = 0,\ p = 3 + \frac{R}{20},\ \ (1, 2).$$

Ans.:

R	Real root	R	Real root
0	1.406539	5	1.371448
1	1.399241	6	1.364827
2	1.392087	7	1.358329
3	1.385074	8	1.351949
4	1.378196	9	1.345683

Problem statement:-

Compute the real root of the following equation lying in $(1,2)$ correct upto **4D** by using **Newton's divided difference inverse interpolation formula** : $f(x) = x^3 - 3.55x^2 + 8x - 7 = 0$.

Working formula:-

To find the root, we take $y = 0$ and this is a problem of inverse interpolation. We first find the values of $f(x) = x^3 - 3.55x^2 + 8x - 7$.
$\therefore \ f(1.39) = -0.053336 < 0$, $f(1.40) = -0.014 < 0$, $f(1.41) = 0.0525466 > 0$, $f(1.42) = 0.065068 > 0$, $f(1.43) = 0.104812 > 0$.
Thus the root lies between 1.39 and 1.43.
Now $f'(x) = 3x^2 - 7.1x + 8 > 0$ for the interval $(1.39, 1.43)$, so in this interval unique root exist.
We find the root by using **Newton's divided difference inverse interpolation formula**. The table is

y	$x = f^{-1}(y) = F(y)$
-0.053336	1.39
-0.014000	1.40
0.025466	1.41
0.065068	1.42
0.104812	1.43

Newton's divided difference inverse interpolation formula is given by

$$x = F(y) \approx F(y_0) + (y - y_0)F[y_1, y_0] + (y - y_0)(y - y_1)F[y_2, y_1, y_0] + \dots$$

$$+(y - y_0)(y - y_1)(y - y_2)\dots(y - y_{n-1})F[y_n, y_{n-1}, \dots, y_1, y_0]$$

$$= F_0 + (y-y_0)\chi F + (y-y_0)(y-y_1)\chi^2 F + \dots + (y-y_0)(y-y_1)(y-y_2)\dots(y-y_{n-1})\chi^n F,$$

where

$$\chi F = F[y_1, y_0] = \frac{F(y_1) - F(y_0)}{y_1 - y_0},$$

$$\chi^2 F = F[y_2, y_1, y_0] = \frac{F[y_1, y_0] - F[y_2, y_1]}{y_2 - y_0},$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\chi^n F = F[y_n, y_{n-1}, \dots, y_1, y_0] = \frac{F[y_n, y_{n-1}, \dots, y_1] - F[y_{n-1}, y_{n-2}, \dots, y_0]}{y_n - y_0}.$$

Result:-

The real root of the equation $f(x) = 0$ correct upto four decimal places is $x \approx 1.4036$.

Computation table for Newton's divided difference inverse interpolation :

				y	$F(y)$	χF	$\chi^2 F$	$\chi^3 F$	$\chi^4 F$
				y_0	$F(y_0)$				
			$y_1 - y_0$			$F[y_1, y_0]$			
		$y_2 - y_0$		y_1	$F(y_1)$		$F[y_2, y_1, y_0]$		
	$y_3 - y_0$		$y_2 - y_1$			$F[y_2, y_1]$		$F[y_3, y_2, y_1, y_0]$	
$y_4 - y_0$		$y_3 - y_1$		y_2	$F(y_2)$		$F[y_3, y_2, y_1]$		$F[y_4, y_3, y_2, y_1, y_0]$
	$y_4 - y_1$		$y_3 - y_2$			$F[y_3, y_2]$		$F[y_4, y_3, y_2, y_1]$	
		$y_4 - y_2$		y_3	$F(y_3)$		$F[y_4, y_3, y_2]$		
			$y_4 - y_3$			$F[y_4, y_3]$			
				y_4	$F(y_4)$				

Computation table for Newton's divided difference inverse interpolation :

				y	$F(y)$	δF	$\delta^2 F$	$\delta^3 F$	$\delta^4 F$
				$-.053336$	1.39				
			.039336			.25422005			
		.078802		$-.014000$	1.4		$-.01062651$		
	.118404		.039466			.25338266		$-.00319837$	
.158148		.079068		.025466	1.41		$-.01100521$		.00079217
	.118812		.039602			.25251250		$-.00307309$	
		.079346		.065068	1.42		$-.01137033$		
			.039744			.25161031			
				.104812	1.43				

Here $y = 0, \quad y_0 = -0.053336$.

Coefficient	Multiplier	Positive term	Negative term
1.0	1.39	1.39	
0.053336	0.25422005	0.0135590806	
0.000746704	−0.01062651		0.000007934858
−0.000019015564	−0.00319837	0.000000060819	
0.000001237305	0.00079217	0.000000000980	
Total :-		1.403559142399	0.000007934858

$$\begin{array}{r} 1.403559142399 \\ -0.000007934858 \\ \hline 1.403551207541 \end{array}$$

$$\therefore \quad F(0) \approx 1.403551207541 \ .$$

NEWTON'S DIVIDED DIFFERENCE INVERSE INTERPOLATION PROBLEMS & ANSWERS

Prob.-1:

$$x^3 - px^2 + 8x - 7 = 0,\ p = 3.5 + \frac{R}{20},\quad (1, 2).$$

Ans.:

R	Real root	R	Real root
0	1.379341	5	1.517788
1	1.403551	6	1.551640
2	1.429331	7	1.588055
3	1.456841	8	1.627284
4	1.486261	9	1.669588

Prob.-2:

$$ax^3 - 18.47x^2 + 12.59x - 19.77 = 0,\ a = 4.4 + \frac{R}{10},\quad (3, 4).$$

Ans.:

R	Real root	R	Real root
0	3.754356	5	3.362102
1	3.668238	6	3.293989
2	3.586206	7	3.228842
3	3.507997	8	3.166487
4	3.433370	9	3.106761

Prob.-3:

$$ax^3 - 2.7x^2 + bx - 5.3 = 0,\ a = 1 + \frac{R}{10},\ b = 7 - a,\quad (1, 2).$$

Ans.:

R	Real root	R	Real root
0	1.266592	5	1.199272
1	1.248911	6	1.190211
2	1.233858	7	1.182042
3	1.220829	8	1.174630
4	1.209402	9	1.167866

Prob.-4:

$$ax^3 - 3.64x^2 + 7.15x - 9.77 = 0,\ a = 1.23 + \frac{R}{100},\quad (1, 3).$$

Ans.:

R	Real root	R	Real root
0	2.025516	5	1.973224
1	2.014616	6	1.963391
2	2.003945	7	1.953752
3	1.993493	8	1.944300
4	1.983255	9	1.935029

Prob.-5:

$$x^3 - ax^2 + bx - 5.33 = 0,\ a = 1.12 + \frac{R}{20},\ b = 3 + a,\quad (1, 2).$$

Ans.:

R	Real root	R	Real root
0	1.246144	5	1.259725
1	1.248735	6	1.262641
2	1.251386	7	1.265630
3	1.254100	8	1.268694
4	1.256879	9	1.271838

Prob.-6:

$$px^3 - 5.43x^2 + 6.49x - 3.72 = 0,\ p = 1.6 + \frac{R}{20},\quad (1, 2).$$

Ans.:

R	Real root	R	Real root
0	1.905275	5	1.484391
1	1.802810	6	1.425049
2	1.709860	7	1.372359
3	1.626132	8	1.325526
4	1.551177	9	1.283795

Prob.-7:

$$bx^3 - 2.7x^2 + ax - 4.2 = 0,\ b = 2.0 + \frac{R}{10},\ a = 3 + b,\quad (0,1).$$

Ans.:

R	Real root	R	Real root
0	0.981953	5	0.840537
1	0.948454	6	0.818428
2	0.917964	7	0.797803
3	0.890040	8	0.778496
4	0.864328	9	0.760365

Prob.-8:

$$px^3 + 1.27x^2 - 4.35x - 5.93 = 0,\ p = 4 + \frac{R}{20},\quad (1,2).$$

Ans.:

R	Real root	R	Real root
0	1.332879	5	1.304928
1	1.327109	6	1.299595
2	1.321431	7	1.294343
3	1.315844	8	1.289170
4	1.310343	9	1.284072

H(III)–Mathematics–H/Pr/8/Module–XVI/Batch–III

2017

MATHEMATICS – HONOURS – PRACTICAL

Eighth Paper

(Module – XVI)

Full Marks – 50

The questions are of equal value

Distribution of Marks :

Three Questions	:	$10 \times 3 = 30$
Viva-Voce	:	10
Sessional	:	10

BATCH–III

Answer **Question No. 1** and **any one** from **Question Nos. 2, 3, 4** and **any one** from **Question Nos. 5, 6**

Throughout the question paper the constant **R** represents the **last digit** of the **Roll No.** of the candidate

1. From the following table calculate $f(x)$ at $x = 5.3 + \frac{R+1}{100}$ and $x = 6.1 - \frac{R+1}{100}$ by suitable interpolation formula :

x	5.3	5.5	5.6	5.8	5.9	6.1
$f(x)$	R.48080	R.66061	R.60655	R.72244	R.59443	R.64479

2. Compute the value of the following integral correct to 4D by Trapezoidal rule and verify the result by Weddle's rule using 13 ordinates :

$$\int_{1.2}^{3.2} \frac{1 + x\ \sinh(1 + bx)}{b + x + x^2}\,dx\ ,\ \text{where } b = \frac{1+R}{40}\ .$$

3. Compute the dominant eigenpair of the following matrix correct to four significant figures by Power method :

$$\begin{pmatrix} 7.71 + c & -10.15 & 4.65 & -3.18 \\ -10.15 & 14.06 + c & -8.14 & 2.23 \\ 4.65 & -8.14 & 5.27 + c & -1.98 \\ -3.18 & 2.23 & -1.98 & 3.52 + c \end{pmatrix}$$

where $c = 2 + \frac{R}{10}$.

4. Solve the following initial value problem by Modified Euler's method to find the values of y for $x = 0.1(0.1)0.5$ correct to 4D.

$$\frac{dy}{dx} = \frac{1 + \cos(dx^3 + fy^3)}{1 + dx^2 + fy^2}$$

with $y(0) = 1.0 + \frac{R}{10}$ and $d = 0.3,\ f = 0.4$.

5. Fit a curve of the form $y = a\ x^b$ to the following data by the method of least squares correct to 4D :

$x =$	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
$y - \frac{R}{10} =$	5.2340	6.0217	6.8311	7.6616	8.5129	9.3847	10.2767	11.1885

6. Write an efficient computer program in Fortran or C to find the smallest positive root of the following equation correct to 6D by Newton-Raphson method :

$$x^x + \alpha \log_{10}(x^2 + 1) = 3$$

where $\alpha = 1 + \frac{R}{10}$.

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H(III)–Mathematics–H/Pr/8/Module–XVI/Batch–III

2017
MATHEMATICS – HONOURS – PRACTICAL
Eighth Paper
(Module – XVI)
Full Marks – 50

BATCH–III (Answer)

1. Interpolation :

R	x	$f(x)$	x	$f(x)$
0	5.31	0.55681	6.09	0.56912
1	5.32	1.61927	6.08	1.50810
2	5.33	2.66959	6.07	2.46023
3	5.34	3.70909	6.06	3.42412
4	5.35	4.73902	6.05	4.39843
5	5.36	5.76054	6.04	5.38192
6	5.37	6.77474	6.03	6.37344
7	5.38	7.78262	6.02	7.37189
8	5.39	8.78512	6.01	8.37627
9	5.40	9.78313	6.00	9.38563

2. Integration :

R	I_T^C	I_W^C
0	1.1435	1.1421
1	1.1935	1.1921
2	1.2457	1.2444
3	1.3003	1.2990
4	1.3574	1.3562
5	1.4173	1.4162
6	1.4802	1.4791
7	1.5463	1.5452
8	1.6158	1.6147
9	1.6889	1.6879

3. Dominant eigen-pair (power method) :

R	Lar.EV	R	Lar.EV
0	28.51	5	29.01
1	28.61	6	29.11
2	28.71	7	29.21
3	28.81	8	29.31
4	28.91	9	29.41

$$\begin{pmatrix} -0.7208 \\ 1.0000 \\ -0.5638 \\ 0.2452 \end{pmatrix}$$

4. Differential equation (Modified Euler's) :

R	$y(0.1)$	$y(0.2)$	$y(0.3)$	$y(0.4)$	$y(0.5)$
0	1.1294	1.2428	1.3403	1.4225	1.4907
1	1.2176	1.3194	1.4059	1.4781	1.5375
2	1.3046	1.3940	1.4691	1.5313	1.5822
3	1.3904	1.4667	1.5302	1.5825	1.6253
4	1.4750	1.5377	1.5896	1.6323	1.6672
5	1.5588	1.6077	1.6481	1.6815	1.7089
6	1.6423	1.6775	1.7069	1.7314	1.7517
7	1.7264	1.7488	1.7678	1.7839	1.7973
8	1.8127	1.8238	1.8335	1.8419	1.8491
9	1.9031	1.9059	1.9086	1.9109	1.9129

5. Curve fitting :

R	Geometric curve
0	$y = (4.5454) \cdot x^{1.5401}$
1	$y = (4.6381) \cdot x^{1.5200}$
2	$y = (4.7309) \cdot x^{1.5005}$
3	$y = (4.8238) \cdot x^{1.4814}$
4	$y = (4.9169) \cdot x^{1.4629}$
5	$y = (5.0101) \cdot x^{1.4448}$
6	$y = (5.1035) \cdot x^{1.4272}$
7	$y = (5.1970) \cdot x^{1.4100}$
8	$y = (5.2906) \cdot x^{1.3932}$
9	$y = (5.3843) \cdot x^{1.3768}$

6. **Root :**

Eq.→	$x^x + \alpha \log_{10}(x^2+1) = 3$	$x^\alpha + \alpha \log_{10}(x^2+1) = 3$
$R\downarrow$	x(Newton-Raphson)	x (Newton-Raphson)
0	1.686689	2.225330
1	1.672238	2.056160
2	1.657712	1.924138
3	1.643119	1.818371
4	1.628471	1.731807
5	1.613776	1.659689
6	1.599045	1.598699
7	1.584290	1.546455
8	1.569520	1.501204
9	1.554748	1.461628