INTERPOLATION WITH EQUALLY SPACED POINTS

(Numerical analysis practical)

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Problem statement:-

Compute the values of $f(x)$ at $x = 0.25$ and at $x = 0.37$ by using **Newton's forward interpolation formula** from the following table :

x	$y = f(x)$
0.20	1.0050041681
0.35	1.0153516187
0.50	1.0314130999
0.65	1.0532789998
0.80	1.0810723718
0.95	1.1149496270
1.10	1.1551014141
1.25	1.2017536930
1.40	1.2551690056
1.55	1.3156479541

Working formula:-

Newton's forward interpolation formula (without error term) is given by

$$f(x) \approx y_0 + u\,\Delta y_0 + \frac{u(u-1)}{2!}\,\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\,\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\,\Delta^4 y_0$$

$$+ \frac{u(u-1)(u-2)(u-3)(u-4)}{5!}\,\Delta^5 y_0 + \ldots + \frac{u(u-1)(u-2)\ldots(u-\overline{n-1})}{n!}\,\Delta^n y_0,$$

where $\Delta y_0 = y_1 - y_0$, $\Delta^2 y_0 = \Delta y_1 - \Delta y_0$, etc. and $u = \frac{x - x_0}{h}$, x_0 be the starting interpolating point and h be the common difference of the nodes.

Results:-

$f(0.25) \approx 1.0078226779$
$f(0.37) \approx 1.0171613620$

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Computation table for Newton's forward interpolation :

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$	$\Delta^7 y$	$\Delta^8 y$	$\Delta^9 y$
x_0	$\boxed{y_0}$									
		$\boxed{\Delta y_0}$								
x_1	y_1		$\boxed{\Delta^2 y_0}$							
		Δy_1		$\boxed{\Delta^3 y_0}$						
x_2	y_2		$\Delta^2 y_1$		$\boxed{\Delta^4 y_0}$					
		Δy_2		$\Delta^3 y_1$		$\boxed{\Delta^5 y_0}$				
x_3	y_3		$\Delta^2 y_2$		$\Delta^4 y_1$		$\boxed{\Delta^6 y_0}$			
		Δy_3		$\Delta^3 y_2$		$\Delta^5 y_1$		$\boxed{\Delta^7 y_0}$		
x_4	y_4		$\Delta^2 y_3$		$\Delta^4 y_2$		$\Delta^6 y_1$		$\boxed{\Delta^8 y_0}$	
		Δy_4		$\Delta^3 y_3$		$\Delta^5 y_2$		$\Delta^7 y_1$		$\boxed{\Delta^9 y_0}$
x_5	y_5		$\Delta^2 y_4$		$\Delta^4 y_3$		$\Delta^6 y_2$		$\Delta^8 y_1$	
		Δy_5		$\Delta^3 y_4$		$\Delta^5 y_3$		$\Delta^7 y_2$		
x_6	y_6		$\Delta^2 y_5$		$\Delta^4 y_4$		$\Delta^6 y_3$			
		Δy_6		$\Delta^3 y_5$		$\Delta^5 y_4$				
x_7	y_7		$\Delta^2 y_6$		$\Delta^4 y_5$					
		Δy_7		$\Delta^3 y_6$						
x_8	y_8		$\Delta^2 y_7$							
		Δy_8								
x_9	y_9									

Here $\Delta y_0 = y_1 - y_0$, $\Delta^2 y_0 = \Delta y_1 - \Delta y_0$, $\ldots$, $\Delta^9 y_0 = \Delta^8 y_1 - \Delta^8 y_0$, etc.

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Computation table for Newton's forward interpolation :

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$	$\Delta^7 y$	$\Delta^8 y$	$\Delta^9 y$
0.20	1.0050041681									
		103474506								
0.35	1.0153516187		57140306							
		160614812		903881						
0.50	1.0314130999		58044187		326653					
		218658999		1230534		6924				
0.65	1.0532789998		59274721		333577		1875			
		277933720		1564111		8799		62		
0.80	1.0810723718		60838832		342376		1937		−27	
		338772552		1906487		10736		35		95
0.95	1.1149496270		62745319		353112		1972		68	
		401517871		2259599		12708		103		
1.10	1.1551014141		65004918		365820		2075			
		466522789		2625419		14783				
1.25	1.2017536930		67630337		380603					
		534153126		3006022						
1.40	1.2551690056		70636359							
		604789485								
1.55	1.3156479541									

Here $x = 0.25,\quad x_0 = 0.20,\quad h = 0.15$. $\therefore\ u = 0.33333333333$.

Coefficient	Multiplier	Positive term	Negative term
1.0	1.0050041681	1.0050041681	
0.3333333333	0.0103474506	0.0034491502	
−0.1111111111	0.0057140306		0.000634892289
0.0617283951	0.0000903881	0.000005579512	
−0.0411522634	0.0000326653		0.000001344351
0.0301783265	0.0000006924	0.000000020895	
−0.0234720317	0.0000001875		0.000000004401
0.0190011685	0.0000000062	0.000000000118	
Total :-		1.008458918826	0.000636240941

$$\begin{array}{r} 1.008458918826 \\ -0.000636240941 \\ \hline 1.007822677885 \end{array}$$

$\therefore\quad f(0.25) \approx 1.0078226779$.

Here $x = 0.37, \quad x_0 = 0.35, \quad h = 0.15$. $\quad \therefore \; u = 0.13333333333$.

Coefficient	Multiplier	Positive term	Negative term
1.0	1.0153516187	1.0153516187	
0.13333333333	0.0160614812	0.002141530826	
−0.05777777778	0.0058044187		0.000335366414
0.03595061728	0.0001230534	0.000004423846	
−0.02576460905	0.0000333577		0.000000859448
0.0199246310	0.0000008799	0.000000017532	
−0.01616108959	0.0000001937		0.000000003130
0.01354453223	0.0000000035	0.000000000047	
Total :-		1.017497590951	0.000336228992

$$\begin{array}{r} 1.017497590951 \\ -0.000336228992 \\ \hline 1.017161361959 \end{array}$$

$\therefore \quad f(0.37) \approx 1.0171613620$.

NEWTON'S FORWARD INTERPOLATION PROBLEMS & ANSWERS

Prob.-1: $x = 0.22$, $x = 0.37$.

x	$f(x)$	x	$f(x)$
0.20	$R + 0.1986693308$	0.80	$R + 0.7173560909$
0.32	$R + 0.3145665606$	0.92	$R + 0.7956016200$
0.44	$R + 0.4259394651$	1.04	$R + 0.8624042272$
0.56	$R + 0.5311861979$	1.16	$R + 0.9168031088$
0.68	$R + 0.6287930240$	1.28	$R + 0.9580158603$

where $R = 0(1)9$.

Ans.: $f(0.22) \approx R + 0.2182296230$ & $f(0.37) \approx R + 0.3616154319$

Prob.-2: $x = 0.20 + \frac{R+1}{100}$, $x = 0.35 + \frac{R+1}{100}$, where $R = 0(1)9$.

x	$h(x)$	x	$h(x)$
0.20	1.0050041681	0.95	1.1149496270
0.35	1.0153516187	1.10	1.1551014141
0.50	1.0314130999	1.25	1.2017536930
0.65	1.0532789998	1.40	1.2551690056
0.80	1.0810723718	1.55	1.3156479541

Ans.:

R	x	$h(x)$	x	$h(x)$
0	0.21	1.0055175665	0.36	1.0162437873
1	0.22	1.0060561029	0.37	1.0171613620
2	0.23	1.0066197908	0.38	1.0181043658
3	0.24	1.0072086442	0.39	1.0190728222
4	0.25	1.0078226779	0.40	1.0200667556
5	0.26	1.0084619072	0.41	1.0210861906
6	0.27	1.0091263481	0.42	1.0221311529
7	0.28	1.0098160172	0.43	1.0232016685
8	0.29	1.0105309317	0.44	1.0242977643
9	0.30	1.0112711096	0.45	1.0254194675

Prob.-3: : $x = 0.02$, $x = 0.08$, where $R = 0(1)9$.

x	$g(x)$	x	$g(x)$
0.00	$R + 1.0099999905$	0.25	$R + 1.0482119882$
0.05	$R + 1.0175281860$	0.30	$R + 1.0560278512$
0.10	$R + 1.0251130553$	0.35	$R + 1.0639025535$
0.15	$R + 1.0327550250$	0.40	$R + 1.0718365381$
0.20	$R + 1.0404545250$	0.45	$R + 1.0798302513$

Ans.: $g(0.02) \approx R + 1.0130044950$ & $g(0.08) \approx R + 1.0220722795$

Prob.-4: $x = 0.50 + \frac{R+1}{100}$, $x = 0.65 + \frac{R+1}{100}$, where $R = 0(1)9$.

x	$f(x)$	x	$f(x)$
0.50	1.4099866960	1.25	1.6956739440
0.65	1.4629870479	1.40	1.7594130672
0.80	1.5171847046	1.55	1.8255480967
0.95	1.5750393702	1.70	1.8941690928
1.10	1.6342439294	1.85	1.9653695011

Ans.:

R	x	$f(x)$	x	$f(x)$
0	0.51	1.4134828033	0.66	1.4664864514
1	0.52	1.4169842320	0.67	1.4700021083
2	0.53	1.4204909820	0.68	1.4735340185
3	0.54	1.4240030534	0.69	1.4770821821
4	0.55	1.4275204461	0.70	1.4806465991
5	0.56	1.4310431602	0.71	1.4842272695
6	0.57	1.4345711956	0.72	1.4878241933
7	0.58	1.4381045524	0.73	1.4914373704
8	0.59	1.4416432306	0.74	1.4950668009
9	0.60	1.4451872301	0.75	1.4987124847

Prob.-5: $x = 0.12,\ x = 0.18.$

x	$h(x)$	x	$h(x)$
0.10	$R + 1.0080320835$	0.35	$R + 1.0283956528$
0.15	$R + 1.0120723248$	0.40	$R + 1.0325175524$
0.20	$R + 1.0161286592$	0.45	$R + 1.0366559029$
0.25	$R + 1.0202013254$	0.50	$R + 1.0408108234$
0.30	$R + 1.0242903233$	0.55	$R + 1.0449823141$

where $R = 0(1)9$.

Ans.: $h(0.12) \approx R + 1.0096462488$ & $h(0.18) \approx R + 1.0145041656$

Prob.-6: $x = 0.20 + \frac{R+1}{100},\ x = 0.35 + \frac{R+1}{100}$, where $R = 0(1)9$.

x	$f(x)$	x	$f(x)$
0.20	1.3096766424	0.95	1.5750393702
0.35	1.3589064140	1.10	1.6342439294
0.50	1.4099866960	1.25	1.6956739440
0.65	1.4629870479	1.40	1.7594130672
0.80	1.5179796437	1.55	1.8255480967

Ans.:

R	x	$f(x)$	x	$f(x)$
0	0.21	1.3129024130	0.36	1.3622534389
1	0.22	1.3161361288	0.37	1.3656087077
2	0.23	1.3193778093	0.38	1.3689722406
3	0.24	1.3226274742	0.39	1.3723440579
4	0.25	1.3258851431	0.40	1.3757241801
5	0.26	1.3291508357	0.41	1.3791126277
6	0.27	1.3324245718	0.42	1.3825094211
7	0.28	1.3357063712	0.43	1.3859145809
8	0.29	1.3389962538	0.44	1.3893281277
9	0.30	1.3422942394	0.45	1.3927500822

Problem statement:-

Compute the values of $f(x)$ at $x = 0.94$ and at $x = 0.97$ by using **Newton's backward interpolation formula** from the following table :

x	$y = f(x)$
0.55	3.0623675585
0.60	3.0682266951
0.65	3.0741181374
0.70	3.0800421238
0.75	3.0859986544
0.80	3.0919880867
0.85	3.0980106592
0.90	3.1040662527
0.95	3.1101553440
1.00	3.1162780523

Working formula:-

Newton's backward interpolation formula (without error term) is given by

$$f(x) \approx y_n + u\,\nabla y_n + \frac{u(u+1)}{2!}\,\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\,\nabla^3 y_n + \frac{u(u+1)(u+2)(u+3)}{4!}\,\nabla^4 y_n$$

$$+ \frac{u(u+1)(u+2)(u+3)(u+4)}{5!}\,\nabla^5 y_n + \ldots + \frac{u(u+1)(u+2)\ldots(u+n-1)}{n!}\,\nabla^n y_n,$$

where $\nabla y_n = y_n - y_{n-1}$, $\nabla^2 y_n = \nabla y_n - \nabla y_{n-1}$, etc. and $u = \frac{x - x_n}{h}$, x_n be the end point of the interpolation and h be the common difference of the arguments.

Results:-

$f(0.94) \approx 3.1089348459$

$f(0.97) \approx 3.1126003933$

Computation table for Newton's backward interpolation :

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$	$\nabla^6 y$	$\nabla^7 y$	$\nabla^8 y$	$\nabla^9 y$
x_0	y_0									
x_1	y_1	∇y_1								
x_2	y_2	∇y_2	$\nabla^2 y_2$							
x_3	y_3	∇y_3	$\nabla^2 y_3$	$\nabla^3 y_3$						
x_4	y_4	∇y_4	$\nabla^2 y_4$	$\nabla^3 y_4$	$\nabla^4 y_4$					
x_5	y_5	∇y_5	$\nabla^2 y_5$	$\nabla^3 y_5$	$\nabla^4 y_5$	$\nabla^5 y_5$				
x_6	y_6	∇y_6	$\nabla^2 y_6$	$\nabla^3 y_6$	$\nabla^4 y_6$	$\nabla^5 y_6$	$\nabla^6 y_6$			
x_7	y_7	∇y_7	$\nabla^2 y_7$	$\nabla^3 y_7$	$\nabla^4 y_7$	$\nabla^5 y_7$	$\nabla^6 y_7$	$\nabla^7 y_7$		
x_8	y_8	∇y_8	$\nabla^2 y_8$	$\nabla^3 y_8$	$\nabla^4 y_8$	$\nabla^5 y_8$	$\nabla^6 y_8$	$\nabla^7 y_8$	$\nabla^8 y_8$	
x_9	y_9	∇y_9	$\nabla^2 y_9$	$\nabla^3 y_9$	$\nabla^4 y_9$	$\nabla^5 y_9$	$\nabla^6 y_9$	$\nabla^7 y_9$	$\nabla^8 y_9$	$\nabla^9 y_9$

Here $\nabla y_1 = y_1 - y_0$, $\nabla^2 y_2 = \nabla y_2 - \nabla y_1$, ..., $\nabla^9 y_9 = \nabla^8 y_9 - \nabla^8 y_8$, etc.

Computation table for Newton's backward interpolation :

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
0.55	3.0623675585			
0.60	3.0682266951	58591366		
0.65	3.0741181374	58914423	323057	
0.70	3.0800421238	59239864	325441	2284
0.75	3.0859986544	59565306	325442	0001
0.80	3.0919880867	59894323	329017	3575
0.85	3.0980106592	60225725	331402	2385
0.90	3.1040662527	60555935	330210	−1192
0.95	3.1101553440	60890913	334978	4768
1.00	3.1162780523	61227083	336170	1192

Here $x = 0.94, \quad x_n = 0.95, \quad h = 0.05$. $\quad \therefore \ u = -0.20$.

Coefficient	Multiplier	Positive term	Negative term
1.00	3.1101553440	3.1101553440	
-0.20	0.0060890913		0.00121781826
-0.08	0.0000334978		0.000002679824
Total :-		3.1101553440	0.001220498084

$$\begin{array}{r} 3.110155344000 \\ -0.001220498084 \\ \hline 3.108934845916 \end{array}$$

$$\therefore \quad f(0.94) \approx 3.1089348459 \ .$$

Again here $x = 0.97, \quad x_n = 1.00, \quad h = 0.05$. $\quad \therefore \ u = -0.60$.

Coefficient	Multiplier	Positive term	Negative term
1.00	3.1162780523	3.1162780523	
-0.60	0.0061227083		0.00367362498
-0.12	0.0000336170		0.00000403404
Total :-		3.1162780523	0.00367765902

$$\begin{array}{r} 3.11627805230 \\ -0.00367765902 \\ \hline 3.11260039328 \end{array}$$

$$\therefore \quad f(0.94) \approx 3.1126003933 \ .$$

NEWTON'S BACKWARD INTERPOLATION PROBLEMS & ANSWERS

Prob.-1: $x = 0.94$, $x = 0.97$.

x	$g(x)$	x	$g(x)$
0.55	$R + 1.0623675585$	0.80	$R + 1.0919880867$
0.60	$R + 1.0682266951$	0.85	$R + 1.0980106592$
0.65	$R + 1.0741181374$	0.90	$R + 1.1040662527$
0.70	$R + 1.0800421238$	0.95	$R + 1.1101553440$
0.75	$R + 1.0859986544$	1.00	$R + 1.1162780523$

where $R = 0(1)9$.

Ans.: $g(0.94) \approx R + 1.1089348459$ & $g(0.97) \approx R + 1.1126003933$

Prob.-2: $x = 1.25 + \frac{R+1}{100}$, $x = 1.40 + \frac{R+1}{100}$, where $R = 0(1)9$.

x	$f(x)$	x	$f(x)$
0.20	1.2922071606	0.95	1.5482135742
0.35	1.3397750591	1.10	1.6052054161
0.50	1.3890939964	1.25	1.6642952050
0.65	1.4402284308	1.40	1.7255601691
0.80	1.4932451930	1.55	1.7890803797

Ans.:

R	x	$f(x)$	x	$f(x)$
0	1.26	1.6683109935	1.41	1.7297237842
1	1.27	1.6723364718	1.42	1.7338974458
2	1.28	1.6763716631	1.43	1.7380811780
3	1.29	1.6804165910	1.44	1.7422750052
4	1.30	1.6844712789	1.45	1.7464789517
5	1.31	1.6885357504	1.46	1.7506930419
6	1.32	1.6926100291	1.47	1.7549173003
7	1.33	1.6966941387	1.48	1.7591517514
8	1.34	1.7007881028	1.49	1.7633964199
9	1.35	1.7048919452	1.50	1.7676513304

Prob.-3: $x = 0.48,\ x = 0.53.$

x	$h(x)$	x	$h(x)$
0.10	$R + 1.0080320835$	0.35	$R + 1.0283956528$
0.15	$R + 1.0120723248$	0.40	$R + 1.0325175524$
0.20	$R + 1.0161286592$	0.45	$R + 1.0366559029$
0.25	$R + 1.0202013254$	0.50	$R + 1.0408108234$
0.30	$R + 1.0242903233$	0.55	$R + 1.0449823141$

where $R = 0(1)9$.

Ans.: $h(0.48) \approx R + 1.0391468668$ & $h(0.53) \approx R + 1.0433117294$

Prob.-4: $x = 1.55 + \frac{R+1}{100},\ x = 1.70 + \frac{R+1}{100}$, where $R = 0(1)9$.

x	$g(x)$	x	$g(x)$
0.50	1.4099866960	1.25	1.6956739440
0.65	1.4629870479	1.40	1.7594130672
0.80	1.5171847046	1.55	1.8255480967
0.95	1.5750393702	1.70	1.8941690928
1.10	1.6342439294	1.85	1.9653695011

Ans.:

R	x	$g(x)$	x	$g(x)$
0	1.56	1.8300454886	1.71	1.8988355383
1	1.57	1.8345539292	1.72	1.9035134479
2	1.58	1.8390734186	1.73	1.9082028215
3	1.59	1.8436039567	1.74	1.9129036592
4	1.60	1.8481455436	1.75	1.9176159609
5	1.61	1.8526981791	1.76	1.9223397267
6	1.62	1.8572618635	1.77	1.9270749565
7	1.63	1.8618365966	1.78	1.9318216504
8	1.64	1.8664223784	1.79	1.9365798083
9	1.65	1.8710192089	1.80	1.9413494303

Prob.-5: $x = 1.25 + \frac{R+1}{100}$, $x = 1.40 + \frac{R+1}{100}$, where $R = 0(1)9$.

x	$f(x)$	x	$f(x)$
0.20	1.3096766424	0.95	1.5750393702
0.35	1.3589064140	1.10	1.6342439294
0.50	1.4099866960	1.25	1.6956739440
0.65	1.4629870479	1.40	1.7594130672
0.80	1.5179796437	1.55	1.8255480967

Ans.:

R	x	$f(x)$	x	$f(x)$
0	1.26	1.6998504369	1.41	1.7637465513
1	1.27	1.7040372166	1.42	1.7680907090
2	1.28	1.7082343084	1.43	1.7724455664
3	1.29	1.7124417379	1.44	1.7768111500
4	1.30	1.7166595303	1.45	1.7811874861
5	1.31	1.7208877113	1.46	1.7855746012
6	1.32	1.7251263064	1.47	1.7899725220
7	1.33	1.7293753413	1.48	1.7943812749
8	1.34	1.7336348416	1.49	1.7988008868
9	1.35	1.7379048333	1.50	1.8032313842

Prob.-6: $x = 5.90 - \frac{R+1}{100}$, $x = 6.00 - \frac{R+1}{100}$, where $R = 0(1)8$.

x	$f(x)$	x	$f(x)$
5.00	0.3765103263	5.60	0.4156923627
5.10	0.3827742822	5.70	0.4226081853
5.20	0.3891424508	5.80	0.4296390656
5.30	0.3956165658	5.90	0.4367869178
5.40	0.4021983898	6.00	0.4440536879
5.50	0.4088897149		

Ans.:

R	x	$f(x)$	x	$f(x)$
0	5.89	0.4360668136	5.99	0.4433216035
1	5.88	0.4353478967	5.98	0.4425907260
2	5.87	0.4346301649	5.97	0.4418610534
3	5.86	0.4339136165	5.96	0.4411325838
4	5.85	0.4331982493	5.95	0.4404053152
5	5.84	0.4324840616	5.94	0.4396792456
6	5.83	0.4317710513	5.93	0.4389543731
7	5.82	0.4310592165	5.92	0.4382306956
8	5.81	0.4303485552	5.91	0.4375082111

Problem statement:-

Compute the values of $f(x)$ at $x = 0.78$ and at $x = 0.92$ by using **Starling's interpolation formula** from the following table :

x	$y = f(x)$
0.15	0.1407825446
0.30	0.1470414286
0.45	0.2131583482
0.60	0.3113257143
0.75	0.4254520089
0.90	0.5550671429
1.05	0.7192278125
1.20	0.9604228571
1.35	1.3484786161
1.50	1.9844642857

Working formula:-

Starling's interpolation formula (without error term) is given by

$$f(x) \approx y_0 + \frac{u}{1!}\,\frac{\Delta y_{-1} + \Delta y_0}{2} + \frac{u^2}{2!}\,\Delta^2 y_{-1} + \frac{u(u^2-1^2)}{3!}\,\frac{\Delta^3 y_{-2} + \Delta^3 y_{-1}}{2} + \frac{u^2(u^2-1^2)}{4!}\,\Delta^4 y_{-2}$$
$$+ \frac{u(u^2-1^2)(u^2-2^2)}{5!}\,\frac{\Delta^5 y_{-3} + \Delta^5 y_{-2}}{2} + \frac{u^2(u^2-1^2)(u^2-2^2)}{6!}\,\Delta^6 y_{-3}$$
$$+ \frac{u(u^2-1^2)(u^2-2^2)(u^2-3^2)}{7!}\,\frac{\Delta^7 y_{-4} + \Delta^7 y_{-3}}{2} + \dots,$$

where $\Delta y_{-1} = y_0 - y_{-1}$, $\Delta y_0 = y_1 - y_0$, etc. and $u = \frac{x - x_0}{h}$, h be the common difference of the nodes.

Results:-

$f(0.78) \approx 0.4498320443$
$f(0.92) \approx 0.5742875100$

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Computation table for Starling's interpolation :

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$	$\Delta^7 y$	$\Delta^8 y$	$\Delta^9 y$
x_{-4}	y_{-4}									
		Δy_{-4}								
x_{-3}	y_{-3}		$\Delta^2 y_{-4}$							
		Δy_{-3}		$\Delta^3 y_{-4}$						
x_{-2}	y_{-2}		$\Delta^2 y_{-3}$		$\Delta^4 y_{-4}$					
		Δy_{-2}		$\Delta^3 y_{-3}$		$\Delta^5 y_{-4}$				
x_{-1}	y_{-1}		$\Delta^2 y_{-2}$		$\Delta^4 y_{-3}$		$\Delta^6 y_{-4}$			
		$\boxed{\Delta y_{-1}}$		$\boxed{\Delta^3 y_{-2}}$		$\boxed{\Delta^5 y_{-3}}$		$\boxed{\Delta^7 y_{-4}}$		
x_0	$\boxed{y_0}$		$\boxed{\Delta^2 y_{-1}}$		$\boxed{\Delta^4 y_{-2}}$		$\boxed{\Delta^6 y_{-3}}$		$\boxed{\Delta^8 y_{-4}}$	
		$\boxed{\Delta y_0}$		$\boxed{\Delta^3 y_{-1}}$		$\boxed{\Delta^5 y_{-2}}$		$\boxed{\Delta^7 y_{-3}}$		$\Delta^9 y_{-4}$
x_1	y_1		$\Delta^2 y_0$		$\Delta^4 y_{-1}$		$\Delta^6 y_{-2}$		$\Delta^8 y_{-3}$	
		Δy_1		$\Delta^3 y_0$		$\Delta^5 y_{-1}$		$\Delta^7 y_{-2}$		
x_2	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$		$\Delta^6 y_{-1}$			
		Δy_2		$\Delta^3 y_1$		$\Delta^5 y_0$				
x_3	y_3		$\Delta^2 y_2$		$\Delta^4 y_1$					
		Δy_3		$\Delta^3 y_2$						
x_4	y_4		$\Delta^2 y_3$							
		Δy_4								
x_5	y_5									

Here $\Delta y_{-4} = y_{-3} - y_{-4}$, $\Delta^2 y_{-4} = \Delta y_{-3} - \Delta y_{-4}$, $\ldots$, $\Delta^9 y_{-4} = \Delta^8 y_{-3} - \Delta^8 y_{-4}$, etc.

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Computation table for Starling's interpolation :

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
0.15	0.1407825446						
		62588840					
0.30	0.1470414186		598580356				
		661169196		−278075891			
0.45	0.2131583482		320504465		117160711		
		981673661		−160915180		39053578	
0.60	0.3113257143		159589285		156214289		−14
		1141262946		− 4700891		39053564	
0.75	0.4254520089		154888394		195267853		15
		1296151340		190566962		39053579	
0.90	0.5550671429		345455356		234321432		−11
		1641606696		424888394		39053568	
1.05	0.7192278125		770343750		273375000		0
		2411950446		698263394		39053568	
1.20	0.9604228571		1468607144		312428568		
		3880557590		1010691962			
1.35	1.3484786161		2479299106				
		6359856696					
1.50	1.9844642857						

Here $x = 0.78, \quad x_0 = 0.75, \quad h = 0.15$. $\quad \therefore \; u = 0.20$.

Coefficient	Multiplier	Positive term	Negative term
1.0	0.4254520089	0.4254520089	
0.2	0.1218707143	0.02437414286	
0.02	0.0154888394	0.000309776788	
-0.032	0.00929330355		0.0002973857136
-0.0016	0.0195267853		0.00003124285648
0.006336	0.00390535715	0.0000247443429	
Total :-		0.4501606728909	0.00032862857008

$$\begin{array}{r} 0.45016067289090 \\ -0.00032862857008 \\ \hline 0.44983204432082 \end{array}$$

$\therefore \quad f(0.78) \approx 0.4498320443$.

Here $x = 0.92, \quad x_0 = 0.90, \quad h = 0.15$. $\quad \therefore \; u = 0.13333333333$.

Coefficient	Multiplier	Positive term	Negative term
1.0	0.5550671429	0.5550671429	
0.13333333333	0.1468879018	0.019585053573	
0.00888888889	0.0345455356	0.0003070714276	
−0.02182716049	0.0307727678		0.0006716821416
−0.00072757202	0.0234321432		0.0000170485717
0.00434603018	0.00390535735	0.0000169728009	
Total :-		0.5749762407015	0.0006887307133

$$\begin{array}{r} 0.5749762407015 \\ -0.0006887307133 \\ \hline 0.5742875099882 \end{array}$$

$$\therefore \quad f(0.92) \approx 0.5742875100 \; .$$

STARLING'S INTERPOLATION PROBLEMS & ANSWERS

Prob.-1: $x = 0.75 + \frac{R+1}{100}$, $x = 0.90 + \frac{R+1}{100}$, where $R = 0(1)9$.

x	$g(x)$	x	$g(x)$
0.15	0.1407825446	0.90	0.5550671429
0.30	0.1470414286	1.05	0.7192278125
0.45	0.2131583482	1.20	0.9604228571
0.60	0.3113257143	1.35	1.3484786161
0.75	0.4254520089	1.50	1.9844642857

Ans.:

R	x	$g(x)$	x	$g(x)$
0	0.76	0.4335133732	0.91	0.5646003479
1	0.77	0.4416390353	0.92	0.5742875100
2	0.78	0.4498320443	0.93	0.5841381583
3	0.79	0.4580958457	0.94	0.5941622954
4	0.80	0.4664342857	0.95	0.6043704018
5	0.81	0.4748516172	0.96	0.6147734419
6	0.82	0.4833525042	0.97	0.6253828682
7	0.83	0.4919420276	0.98	0.6362106272
8	0.84	0.5006256896	0.99	0.6472691643
9	0.85	0.5094094197	1.00	0.6585714286

Prob.-2: $x = 1.83$, $x = 1.87$.

x	$f(x)$	x	$f(x)$
1.60	$R + 0.1365530491$	1.85	$R + 0.1595128775$
1.65	$R + 0.1411082745$	1.90	$R + 0.1641602516$
1.70	$R + 0.1456818581$	1.95	$R + 0.1688262224$
1.75	$R + 0.1502737999$	2.00	$R + 0.1735109091$
1.80	$R + 0.1548841000$	2.05	$R + 0.1782143116$

where $R = 0(1)9$.

Ans.: $f(1.83) \approx R + 0.1576591416$ & $f(1.87) \approx R + 0.1613695922$

Prob.-3: $x = 0.65 + \frac{R+1}{100}$, $x = 0.80 + \frac{R+1}{100}$, where $R = 0(1)9$.

x	$g(x)$	x	$g(x)$
0.20	1.3831084535	0.95	1.7464470528
0.35	1.4491591166	1.10	1.8298490344
0.50	1.5183640443	1.25	1.9172338970
0.65	1.5908738693	1.40	2.0087918438
0.80	1.6668464178	1.55	2.1047221614

where $R = 0(1)9$.

Ans.:

R	x	$g(x)$	x	$g(x)$
0	0.66	1.5958291886	0.81	1.6720383795
1	0.67	1.6007999429	0.82	1.6772465133
2	0.68	1.6057861803	0.83	1.6824708696
3	0.69	1.6107879490	0.84	1.6877114990
4	0.70	1.6158052975	0.85	1.6929684521
5	0.71	1.6208382742	0.86	1.6982417797
6	0.72	1.6258869278	0.87	1.7035315329
7	0.73	1.6309513072	0.88	1.7088377629
8	0.74	1.6360314613	0.89	1.7141605210
9	0.75	1.6411274393	0.90	1.7194998586

Prob.-4: $x = 0.80 + \frac{R+1}{100}$, $x = 0.95 + \frac{R+1}{100}$, where $R = 0(1)9$.

x	$h(x)$	x	$h(x)$
0.20	1.3096766424	0.95	1.5750393702
0.35	1.3589064140	1.10	1.6342439294
0.50	1.4099866960	1.25	1.6956739440
0.65	1.4629870479	1.40	1.7594130672
0.80	1.5179796437	1.55	1.8255480967

Ans.:

R	x	$h(x)$	x	$h(x)$
0	0.81	1.5217184705	0.96	1.5789187367
1	0.82	1.5254665061	0.97	1.5828076582
2	0.83	1.5292237733	0.98	1.5867061582
3	0.84	1.5329902947	0.99	1.5906142604
4	0.85	1.5367660931	1.00	1.5945319883
5	0.86	1.5405511915	1.01	1.5984593657
6	0.87	1.5443456126	1.02	1.6023964163
7	0.88	1.5481493795	1.03	1.6063431640
8	0.89	1.5519625153	1.04	1.6102996326
9	0.90	1.5557850428	1.05	1.6142658462

Prob.-5: $x = 3.10 + \frac{R+1}{100}$, $x = 3.25 + \frac{R+1}{100}$, where $R = 0(1)9$.

x	$f(x)$	x	$f(x)$
2.50	0.2492472745	3.25	0.2820813414
2.65	0.2554931180	3.40	0.2891499680
2.80	0.2618954750	3.55	0.2963957261
2.95	0.2684582676	3.70	0.3038230544
3.10	0.2751855160	3.85	0.3114365027

Ans.:

R	x	$f(x)$	x	$f(x)$
0	3.11	0.2756399469	3.26	0.2825471598
1	3.12	0.2760951282	3.27	0.2830137475
2	3.13	0.2765510612	3.28	0.2834811056
3	3.14	0.2770077472	3.29	0.2839492355
4	3.15	0.2774651872	3.30	0.2844181385
5	3.16	0.2779233827	3.31	0.2848878158
6	3.17	0.2783823348	3.32	0.2853582687
7	3.18	0.2788420448	3.33	0.2858294985
8	3.19	0.2793025140	3.34	0.2863015065
9	3.20	0.2797637435	3.35	0.2867742939

Problem statement:-

Compute the values of $g(x)$ at $x = 0.43$ and at $x = 0.47$ by using **Bessel's interpolation formula** from the following table :

x	$y = g(x)$
0.20	4.3403906822
0.25	4.4147616625
0.30	4.4932591915
0.35	4.5761120319
0.40	4.6635620594
0.45	4.7558640242
0.50	4.8532874584
0.55	4.9561164379
0.60	5.0646507740
0.65	5.1792070866

Working formula:-

Bessel's interpolation formula (without error term) is given by

$$g(x) \approx \frac{y_0 + y_1}{2} + \frac{v}{1!}\,\Delta y_0 + \frac{v^2 - \frac{1}{4}}{2!}\,\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2} + \frac{v\left(v^2 - \frac{1}{4}\right)}{3!}\,\Delta^3 y_{-1}$$
$$+ \frac{\left(v^2 - \frac{1}{4}\right)\left(v^2 - \frac{9}{4}\right)}{4!}\,\frac{\Delta^4 y_{-2} + \Delta^4 y_{-1}}{2} + \frac{v\left(v^2 - \frac{1}{4}\right)\left(v^2 - \frac{9}{4}\right)}{5!}\,\Delta^5 y_{-2}$$
$$+ \frac{\left(v^2 - \frac{1}{4}\right)\left(v^2 - \frac{9}{4}\right)\left(v^2 - \frac{25}{4}\right)}{6!}\,\frac{\Delta^6 y_{-3} + \Delta^6 y_{-2}}{2} + \frac{v\left(v^2 - \frac{1}{4}\right)\left(v^2 - \frac{9}{4}\right)\left(v^2 - \frac{25}{4}\right)}{7!}\,\Delta^7 y_{-3} + \ldots,$$

where $\Delta y_{-1} = y_0 - y_{-1}$, $\Delta y_0 = y_1 - y_0$, etc. and $v = u - \frac{1}{2}$ and $u = \frac{x - x_0}{h}$, h be the common difference of the arguments.

Results:-

$g(0.43) \approx 4.7183440842$
$g(0.47) \approx 4.7942032524$

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Computation table for Bessel's interpolation :

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$	$\Delta^7 y$	$\Delta^8 y$	$\Delta^9 y$
x_{-4}	y_{-4}									
		Δy_{-4}								
x_{-3}	y_{-3}		$\Delta^2 y_{-4}$							
		Δy_{-3}		$\Delta^3 y_{-4}$						
x_{-2}	y_{-2}		$\Delta^2 y_{-3}$		$\Delta^4 y_{-4}$					
		Δy_{-2}		$\Delta^3 y_{-3}$		$\Delta^5 y_{-4}$				
x_{-1}	y_{-1}		$\Delta^2 y_{-2}$		$\Delta^4 y_{-3}$		$\Delta^6 y_{-4}$			
		Δy_{-1}		$\Delta^3 y_{-2}$		$\Delta^5 y_{-3}$		$\Delta^7 y_{-4}$		
x_0	y_0		$\Delta^2 y_{-1}$		$\Delta^4 y_{-2}$		$\Delta^6 y_{-3}$		$\Delta^8 y_{-4}$	
		Δy_0		$\Delta^3 y_{-1}$		$\Delta^5 y_{-2}$		$\Delta^7 y_{-3}$		$\Delta^9 y_{-4}$
x_1	y_1		$\Delta^2 y_0$		$\Delta^4 y_{-1}$		$\Delta^6 y_{-2}$		$\Delta^8 y_{-3}$	
		Δy_1		$\Delta^3 y_0$		$\Delta^5 y_{-1}$		$\Delta^7 y_{-2}$		
x_2	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$		$\Delta^6 y_{-1}$			
		Δy_2		$\Delta^3 y_1$		$\Delta^5 y_0$				
x_3	y_3		$\Delta^2 y_2$		$\Delta^4 y_1$					
		Δy_3		$\Delta^3 y_2$						
x_4	y_4		$\Delta^2 y_3$							
		Δy_4								
x_5	y_5									

Computation table for Bessel's interpolation :

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
0.20	4.3403906822					
		743709803				
0.25	4.4147616625		41265487			
		784975290		2287627		
0.30	4.4932591915		43553114		131130	
		828528404		2418757		−2385
0.35	4.5761120319		45971871		128745	
		874500275		2547502		19074
0.40	4.6635620594		48519373		147819	
		923019648		2695321		−2381
0.45	4.7558640242		51214694		145438	
		974234342		2840759		11916
0.50	4.8532874584		54055453		157354	
		1028289795		2998113		10732
0.55	4.9561164379		57053566		168086	
		1085343361		3166199		
0.60	5.0646507740		60219765			
		1145563126				
0.65	5.1792070866					

Here $x = 0.43, \quad x_0 = 0.40, \quad h = 0.05$. $\quad \therefore \; u = 0.60$ & $v = 0.10$.

Coefficient	Multiplier	Positive term	Negative term
1.0	4.7097130418	4.7097130418	
0.1	0.0923019648	0.00923019648	
-0.12	0.00498670335		0.000598404402
-0.004	0.0002695321		0.0000010781284
0.0224	0.00001466285	0.0000003284478	
Total :-		4.7189435667278	0.0005994825304

$$\begin{array}{r} 4.71894356672784 \\ -0.00059948253040 \\ \hline 4.71834408419744 \end{array}$$

$\therefore \quad g(0.43) \approx 4.7183440842$.

Here $x = 0.47, \quad x_0 = 0.45, \quad h = 0.05$. $\quad \therefore \; u = 0.40$ & $v = -0.10$.

Coefficient	Multiplier	Positive term	Negative term
1.0	4.8045757413	4.8045757413	
-0.1	0.0974234342		0.00974234342
-0.12	0.00526350735		0.000631620882
0.004	0.0002840759	0.0000011363036	
0.0224	0.0000151396	0.000000339127	
Total :-		4.8045772167306	0.010373964302

$$\begin{array}{r} 4.8045772167306 \\ -0.0103739643020 \\ \hline 4.7942032524286 \end{array}$$

$$\therefore \quad g(0.47) \approx 4.7942032524 \; .$$

BESSEL'S INTERPOLATION PROBLEMS & ANSWERS

Prob.-1: $x = 0.43$, $x = 0.47$.

x	$h(x)$	x	$h(x)$
0.20	$R + 1.3403906822$	0.45	$R + 1.7558640242$
0.25	$R + 1.4147616625$	0.50	$R + 1.8532874584$
0.30	$R + 1.4932591915$	0.55	$R + 1.9561164379$
0.35	$R + 1.5761120319$	0.60	$R + 2.0646507740$
0.40	$R + 1.6635620594$	0.65	$R + 2.1792070866$

where $R = 0(1)9$.

Ans.: $h(0.43) \approx R + 1.7183440842$ & $h(0.47) \approx R + 1.7942032524$

Prob.-2: $x = 3.10 + \frac{R+1}{100}$, $x = 3.25 + \frac{R+1}{100}$, where $R = 0(1)9$.

x	$f(x)$	x	$f(x)$
2.50	$R + 0.1220327806$	3.25	$R + 0.1784736860$
2.65	$R + 0.1332027480$	3.40	$R + 0.1899407474$
2.80	$R + 0.1444315119$	3.55	$R + 0.2014681691$
2.95	$R + 0.1557193819$	3.70	$R + 0.2130562689$
3.10	$R + 0.1670666690$	3.85	$R + 0.2247053663$

Ans.:

R	x	$f(x)$	x	$f(x)$
0	3.11	0.1678252751	3.26	0.1792362852
1	3.12	1.1685841467	3.27	1.1799991514
2	3.13	2.1693432840	3.28	2.1807622846
3	3.14	3.1701026870	3.29	3.1815256850
4	3.15	4.1708623558	3.30	4.1822893526
5	3.16	5.1716222906	3.31	5.1830532876
6	3.17	6.1723824914	3.32	6.1838174899
7	3.18	7.1731429584	3.33	7.1845819598
8	3.19	8.1739036915	3.34	8.1853466973
9	3.20	9.1746646910	3.35	9.1861117025

Prob.-3: $x = 1.83,\ x = 1.87.$

x	$g(x)$	x	$g(x)$
1.60	$R + 0.1365530491$	1.85	$R + 0.1595128775$
1.65	$R + 0.1411082745$	1.90	$R + 0.1641602516$
1.70	$R + 0.1456818581$	1.95	$R + 0.1688262224$
1.75	$R + 0.1502737999$	2.00	$R + 0.1735109091$
1.80	$R + 0.1548841000$	2.05	$R + 0.1782143116$

where $R = 0(1)9$.

Ans.: $g(1.83) \approx R + 0.1576591421$ & $g(1.87) \approx R + 0.1613695955$

Prob.-4: $x = 0.75 + \frac{R+1}{100},\ x = 0.90 + \frac{R+1}{100}$, where $R = 0(1)9$.

x	$h(x)$	x	$h(x)$
0.15	0.1407825446	0.90	0.5550671429
0.30	0.1470414286	1.05	0.7192278125
0.45	0.2131583482	1.20	0.9604228571
0.60	0.3113257143	1.35	1.3484786161
0.75	0.4254520089	1.50	1.9844642857

Ans.:

R	x	$h(x)$	x	$h(x)$
0	0.76	0.4335133732	0.91	0.5646003479
1	0.77	0.4416390353	0.92	0.5742875100
2	0.78	0.4498320443	0.93	0.5841381583
3	0.79	0.4580958457	0.94	0.5941622954
4	0.80	0.4664342857	0.95	0.6043704018
5	0.81	0.4748516172	0.96	0.6147734419
6	0.82	0.4833525042	0.97	0.6253828682
7	0.83	0.4919420276	0.98	0.6362106272
8	0.84	0.5006256896	0.99	0.6472691643
9	0.85	0.5094094197	1.00	0.6585714286

Prob.-5: $x = 2.22,\ x = 2.28.$

x	$f(x)$	x	$f(x)$
2.00	$R + 0.8589280844$	2.25	$R + 1.0087246895$
2.05	$R + 0.8879659176$	2.30	$R + 1.0401024818$
2.10	$R + 0.9174573421$	2.35	$R + 1.0719702244$
2.15	$R + 0.9474093914$	2.40	$R + 1.1043360233$
2.20	$R + 0.9778294563$	2.45	$R + 1.1372072697$

where $R = 0(1)9$.

Ans.: $f(2.22) \approx R + 0.9901301155$ & $f(2.28) \approx R + 1.0274929848$

Prob.-6: $x = 1.00 + \frac{R+1}{100},\ x = 1.15 + \frac{R+1}{100}$, where $R = 0(1)9$.

x	$g(x)$	x	$g(x)$
0.40	0.1536586744	1.15	0.1713115352
0.55	0.1570373606	1.30	0.1750783770
0.70	0.1604903382	1.45	0.1789280451
0.85	0.1640192407	1.60	0.1828623607
1.00	0.1676257377	1.75	0.1868831850

Ans.:

R	x	$g(x)$	x	$g(x)$
0	1.01	0.1678689713	1.16	0.1715601171
1	1.02	0.1681125579	1.17	0.1718090597
2	1.03	0.1683564979	1.18	0.1720583635
3	1.04	0.1686007919	1.19	0.1723080291
4	1.05	0.1688454404	1.20	0.1725580570
5	1.06	0.1690904438	1.21	0.1728084477
6	1.07	0.1693358028	1.22	0.1730592017
7	1.08	0.1695815178	1.23	0.1733103195
8	1.09	0.1698275894	1.24	0.1735618018
9	1.10	0.1700740180	1.25	0.1738136489

H(III)−Mathematics−H/Pr/8 (Module−XVI)/Batch−1

2017
MATHEMATICS – HONOURS – PRACTICAL
Eighth Paper
(Module – XVI)
Full Marks – 50

The questions are of equal value

Distribution of Marks :

Three Questions	:	$10 \times 3 = 30$
Viva-Voce	:	10
Sessional	:	10

BATCH–I

Answer **Question No. 1** and **any one** from **Question Nos. 2, 3, 4** and **any one** from **Question Nos. 5, 6**

Throughout the question paper the constant **R** represents the **last digit** of the **Roll No.** of the candidate

1. Compute the values of $f(x)$ at $x = 0.30 + \frac{R+1}{100}$ and $x = 1.50 + \frac{R+1}{100}$ from the following table using suitable interpolation formula :

x	$f(x)$
0.30	1.3422942394
0.45	1.3927500822
0.60	1.4451025226
0.75	1.4994228523
0.90	1.5557850428
1.05	1.6142658462
1.20	1.6749448995
1.35	1.7379048333
1.50	1.8032313843
1.65	1.8710135117

2. Compute the value of the following integral correct to 4D by Trapezoidal rule and verify the result by Weddle's rule taking 13 ordinates :

$$\int_{20^\circ}^{40^\circ} \frac{dx}{(1 - a \sin^2 x)^{\frac{3}{2}}} \text{ , where } a = \frac{1+R}{20} .$$

3. Solve the following system of linear equations by Gauss-Elimination method correct to 6D :

$$AX = B,$$

where $A = \begin{pmatrix} 3.10+b & 1.21 & 0.80 & -1.70 \\ 2.20 & 4.44+b & 1.00 & 0.34 \\ 1.58 & -0.34 & 3.26+b & 0.95 \\ 1.97 & 1.08 & -2.07 & 5.14+b \end{pmatrix}$

where $b = 2 + \frac{R}{10}$,
$X = (x_1,\ x_2,\ x_3,\ x_4)^T$,
$B = (-1.51,\ 8.21, -8.56,\ 30.97)^T$.

4. Compute the smallest positive root of the following equation correct up to 2D by Bisection method and improve it correct to 5D by fixed point iteration :

$$x^{x-\cos x} - x\ \sin x - c = 0\ ,\ \text{where } c = 2 + \frac{R}{10}\ .$$

5. Fit a curve of the form $y = a + bx + cx^2$ to the following data using least square method correct to 4D :

x	1.2	2.2	3.2	4.2	5.2	6.2	7.2	8.2
y	$3.5+d$	$5.5+d$	$8.3+d$	$11.1+d$	$14.3+d$	$18.5+d$	$22.1+d$	$27.3+d$

where $d = \frac{1+R}{20}$.

6. Write an efficient computer program in C or Fortran to solve the following initial value problem by 4^{th} order Runge-Kutta method correct to 5D :

$$\frac{dy}{dx} = \frac{1+\sin(\alpha xy)}{1+e^{\alpha xy^2}} \ \text{ with } y(0) = 1 + \frac{R}{10}\ ,\ \alpha = 1 + \frac{R}{10}\ .$$

The output will show the values of y for $x = 0.1(0.1)1.0$.

H(III)−Mathematics−H/Pr/8 (Module−XVI)/Batch−1

2017
MATHEMATICS – HONOURS – PRACTICAL
Eighth Paper
(Module – XVI)
Full Marks – 50

BATCH–I (Answer)

1. Interpolation :

R	x	$f(x)$	x	$f(x)$
0	0.31	1.3456003481	1.51	1.8076727942
1	0.32	1.3489145998	1.52	1.8121251434
2	0.33	1.3522370146	1.53	1.8165884589
3	0.34	1.3555676126	1.54	1.8210627677
4	0.35	1.3589064140	1.55	1.8255480968
5	0.36	1.3622534389	1.56	1.8300444734
6	0.37	1.3656087077	1.57	1.8345519247
7	0.38	1.3689722406	1.58	1.8390704779
8	0.39	1.3723440579	1.59	1.8436001605
9	0.40	1.3757241801	1.60	1.8481409998

2. Integration :

R	I_T^C	I_W^C
0	0.3559	0.3559
1	0.3629	0.3629
2	0.3702	0.3702
3	0.3778	0.3778
4	0.3857	0.3857
5	0.3939	0.3939
6	0.4025	0.4025
7	0.4114	0.4114
8	0.4207	0.4206
9	0.4304	0.4303

3. Gauss-elimination :

R	x_1	x_2	x_3	x_4
0	0.8748	1.1779	−2.3962	3.2233
1	0.8471	1.1618	−2.3410	3.2045
2	0.8205	1.1461	−2.2880	3.1852
3	0.7950	1.1309	−2.2372	3.1655
4	0.7705	1.1161	−2.1884	3.1455
5	0.7469	1.1017	−2.1414	3.1251
6	0.7243	1.0877	−2.0962	3.1045
7	0.7026	1.0740	−2.0528	3.0838
8	0.6817	1.0607	−2.0109	3.0628
9	0.6616	1.0478	−1.9706	3.0418

4. Root :

R	x(Bisection)	x (Fixed point)
0	0.31	0.31335
1	0.30	0.30143
2	0.29	0.29042
3	0.28	0.28022
4	0.27	0.27075
5	0.26	0.26194
6	0.25	0.25370
7	0.25	0.24600
8	0.24	0.23878
9	0.23	0.23200

5. Curve fitting :

R	Parabolic curve
0	$y = 1.6066 + 1.3707\ x + 0.2131\ x^2$
1	$y = 1.6566 + 1.3707\ x + 0.2131\ x^2$
2	$y = 1.7066 + 1.3707\ x + 0.2131\ x^2$
3	$y = 1.7566 + 1.3707\ x + 0.2131\ x^2$
4	$y = 1.8066 + 1.3707\ x + 0.2131\ x^2$
5	$y = 1.8566 + 1.3707\ x + 0.2131\ x^2$
6	$y = 1.9066 + 1.3707\ x + 0.2131\ x^2$
7	$y = 1.9566 + 1.3707\ x + 0.2131\ x^2$
8	$y = 2.0066 + 1.3707\ x + 0.2131\ x^2$
9	$y = 2.0566 + 1.3707\ x + 0.2131\ x^2$

6. Differential equation (RK4) :

$\downarrow y/R \rightarrow$	0	1	2	3	4
$y(0.1)$	1.05115	1.15120	1.25119	1.35110	1.45093
$y(0.2)$	1.10412	1.20399	1.30354	1.40268	1.50135
$y(0.3)$	1.15787	1.25684	1.35487	1.45180	1.54754
$y(0.4)$	1.21106	1.30787	1.40278	1.49568	1.58658
$y(0.5)$	1.26209	1.35513	1.44520	1.53240	1.61714
$y(0.6)$	1.30939	1.39704	1.48083	1.56136	1.63956
$y(0.7)$	1.35165	1.43263	1.50938	1.58309	1.65518
$y(0.8)$	1.38810	1.46172	1.53136	1.59875	1.66563
$y(0.9)$	1.41854	1.48473	1.54774	1.60967	1.67238
$y(1.0)$	1.44327	1.50245	1.55964	1.61708	1.67661

$\downarrow y/R \rightarrow$	5	6	7	8	9
$y(0.1)$	1.55066	1.65028	1.74976	1.84910	1.94829
$y(0.2)$	1.59949	1.69706	1.79404	1.89042	1.98626
$y(0.3)$	1.64206	1.73540	1.82768	1.91913	2.01001
$y(0.4)$	1.67568	1.76334	1.85002	1.93625	2.02257
$y(0.5)$	1.70003	1.78185	1.86340	1.94541	2.02847
$y(0.6)$	1.71649	1.79326	1.87082	1.94992	2.03101
$y(0.7)$	1.72705	1.79991	1.87470	1.95199	2.03201
$y(0.8)$	1.73352	1.80360	1.87661	1.95287	2.03237
$y(0.9)$	1.73735	1.80556	1.87750	1.95322	2.03249
$y(1.0)$	1.73952	1.80654	1.87788	1.95334	2.03252