POLARIZATION

Syllabus:

Different states of polarisation; double refraction, for uniaxial crystals; polaroids and their uses. Production and analysis of plane, circularly and Huygen's construction-elliptically polarised light by retardation plates and rotatory polarisation and optical activity; Fresnel's explanation of optical activity; Biquartz and half shade polarimeter

References:

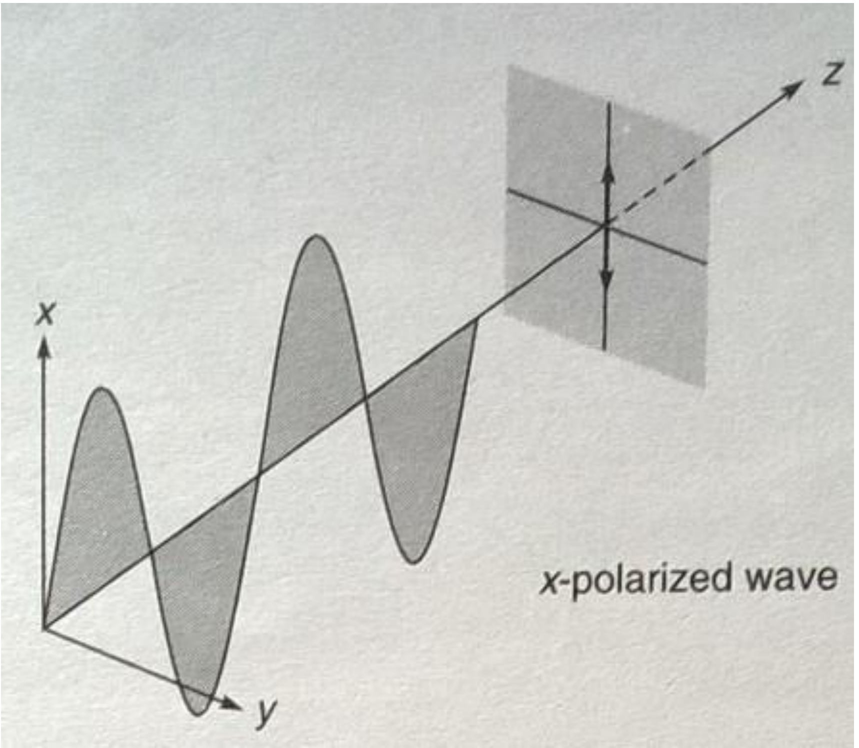
1) A Text Book on Light – B. Ghosh & K. G. Mazumdar (Sreedhar Publishers)
2) Optics – Ajoy Ghatak (Tata McGraw-Hill)
3) Modern Optics – A. B. Gupta (Books and Allied)

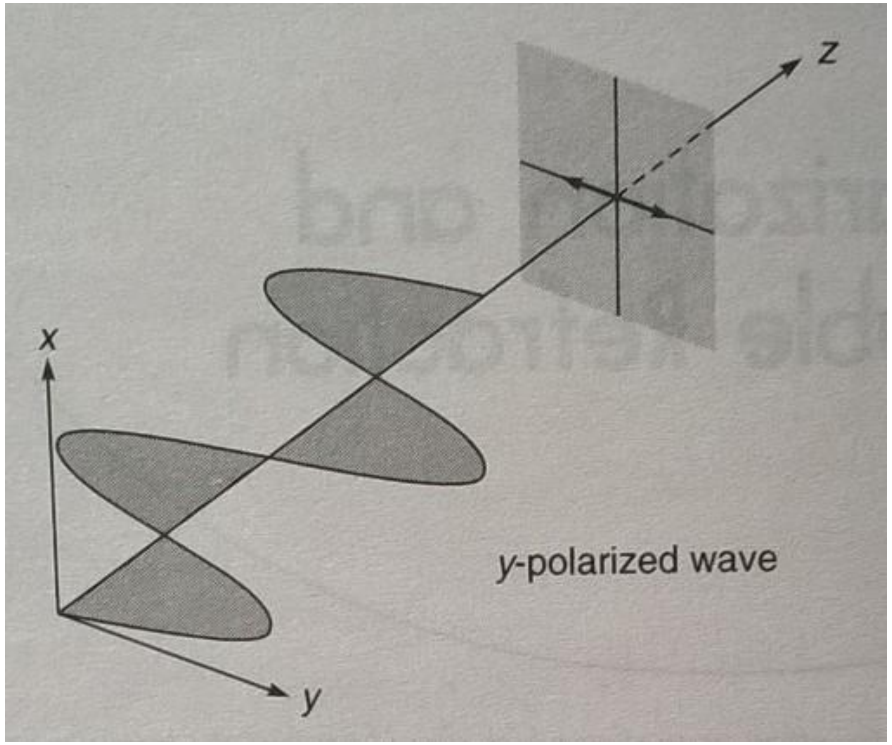
TRANSVERSE VIBRATION IN STRING

Linearly Polarized Wave:

$$x(z,t) = a\cos(kz - \omega t)$$
$$y(z,t) = 0$$

$$y(z,t) = a\cos(kz - \omega t)$$
$$x(z,t) = 0$$



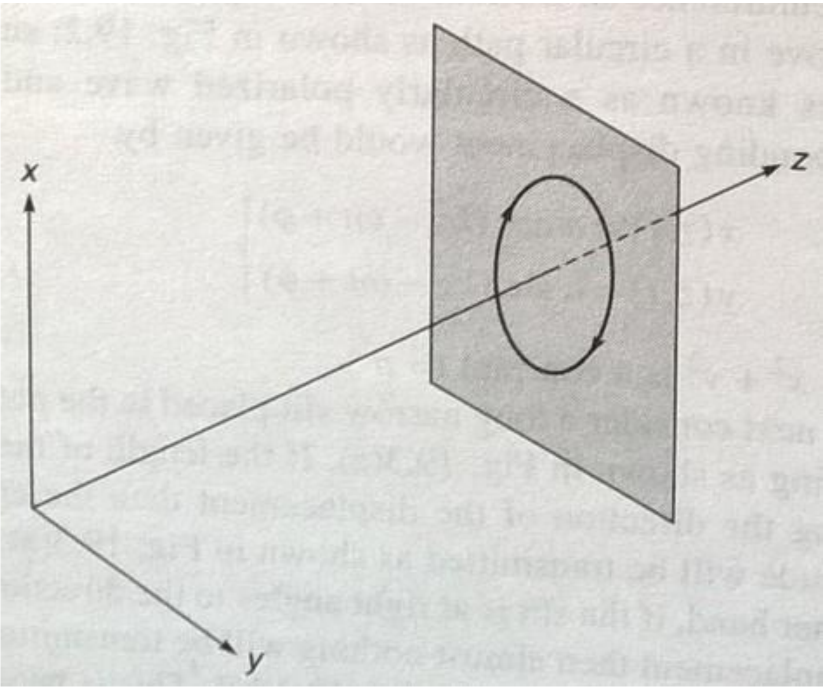


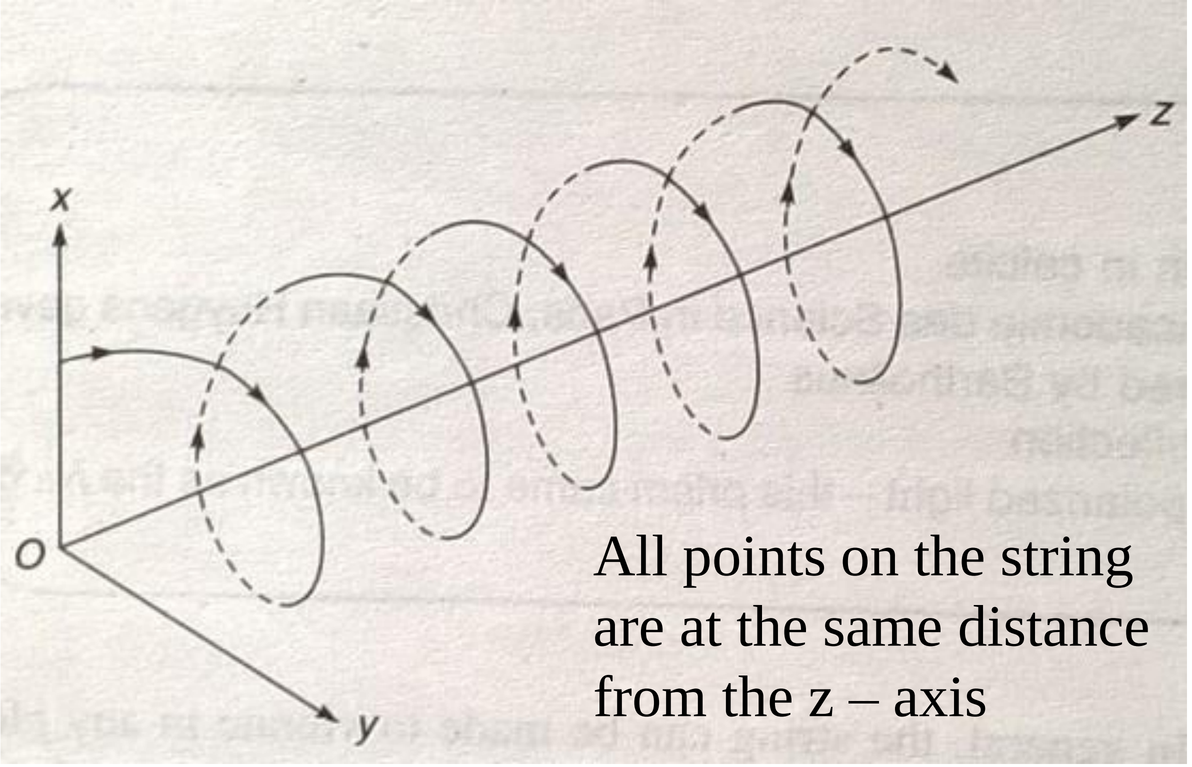
TRANSVERSE VIBRATION IN STRING

Circular Polarization:

$$x(z,t) = a\cos(kz - \omega t)$$

$$y(z,t) = a\sin(kz - \omega t)$$





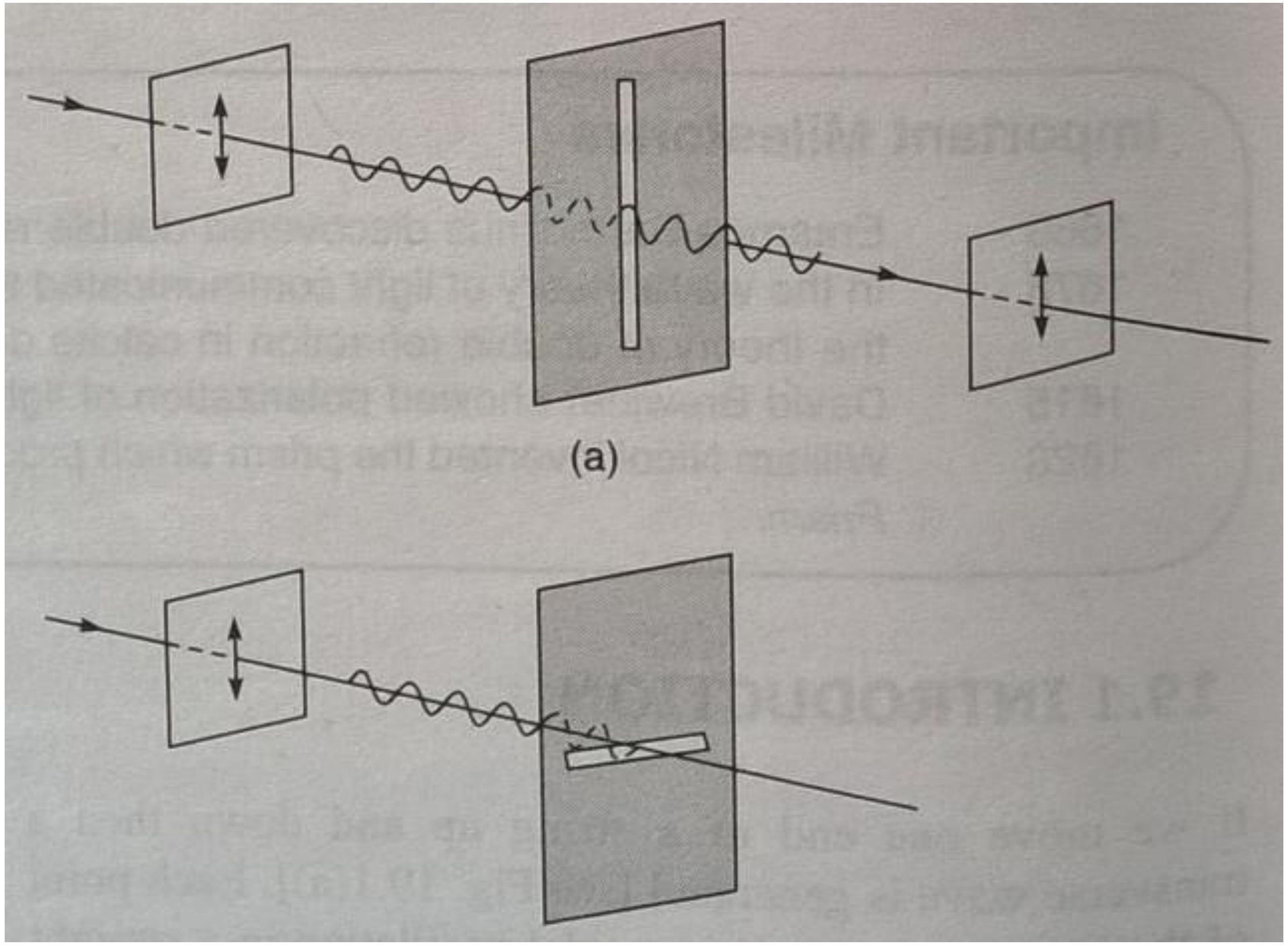
All points on the string are at the same distance from the z – axis

Each point on the string rotates on the circumference of the circle

TRANSVERSE VIBRATION IN STRING

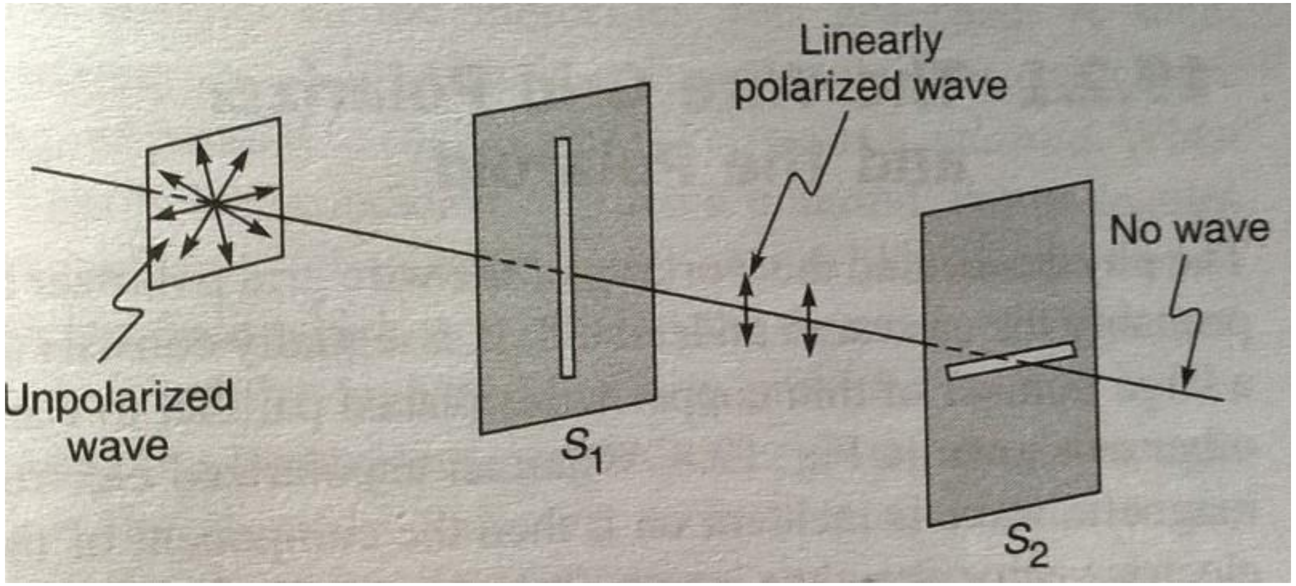
Linearly polarized transverse wave (propagating through the string) incident on a narrow slit:

Slit allows only the component of the displacement, which is along the length of the slit, to pass through



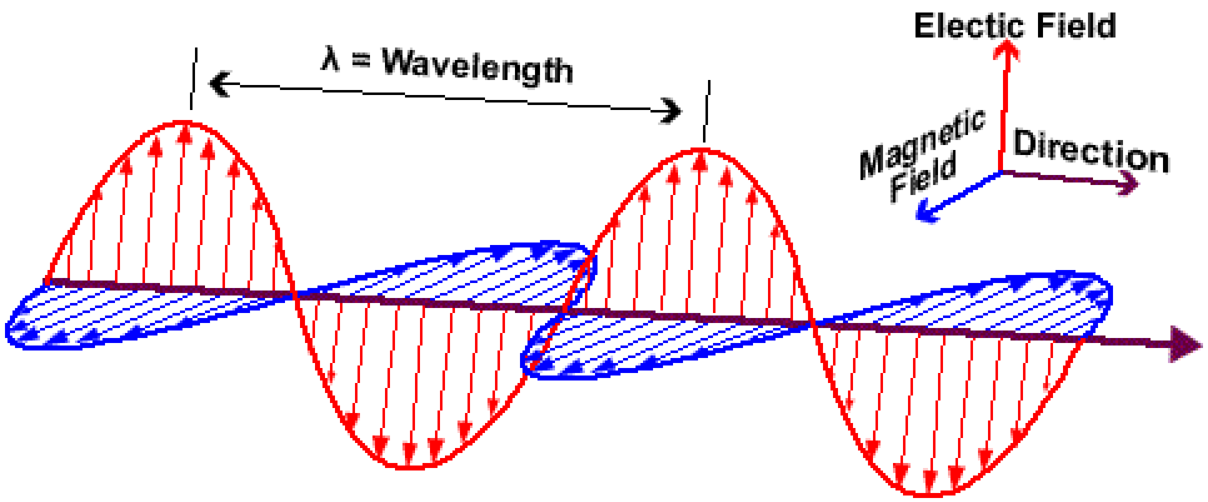
TRANSVERSE VIBRATION IN STRING

- Unpolarized transverse wave (propagating through the string) incident on a narrow slit S_1.
- The transmitted beam will be linearly polarized and its amplitude will not depend on the orientation of S_1.
- If this polarized wave is allowed to pass through another narrow slit S_2, then the intensity of the emergent wave will depend on the relative orientation of S_2 with respect to S_1.

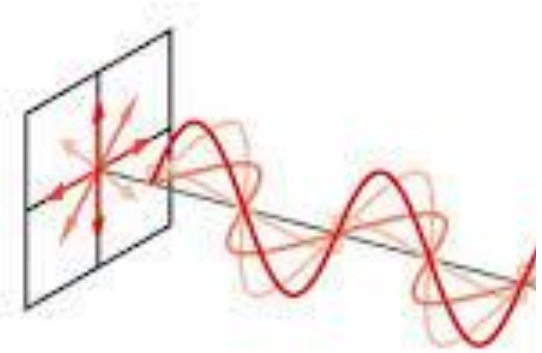


ELECTROMAGNETIC WAVE

Electromagnetic wave – a transverse wave



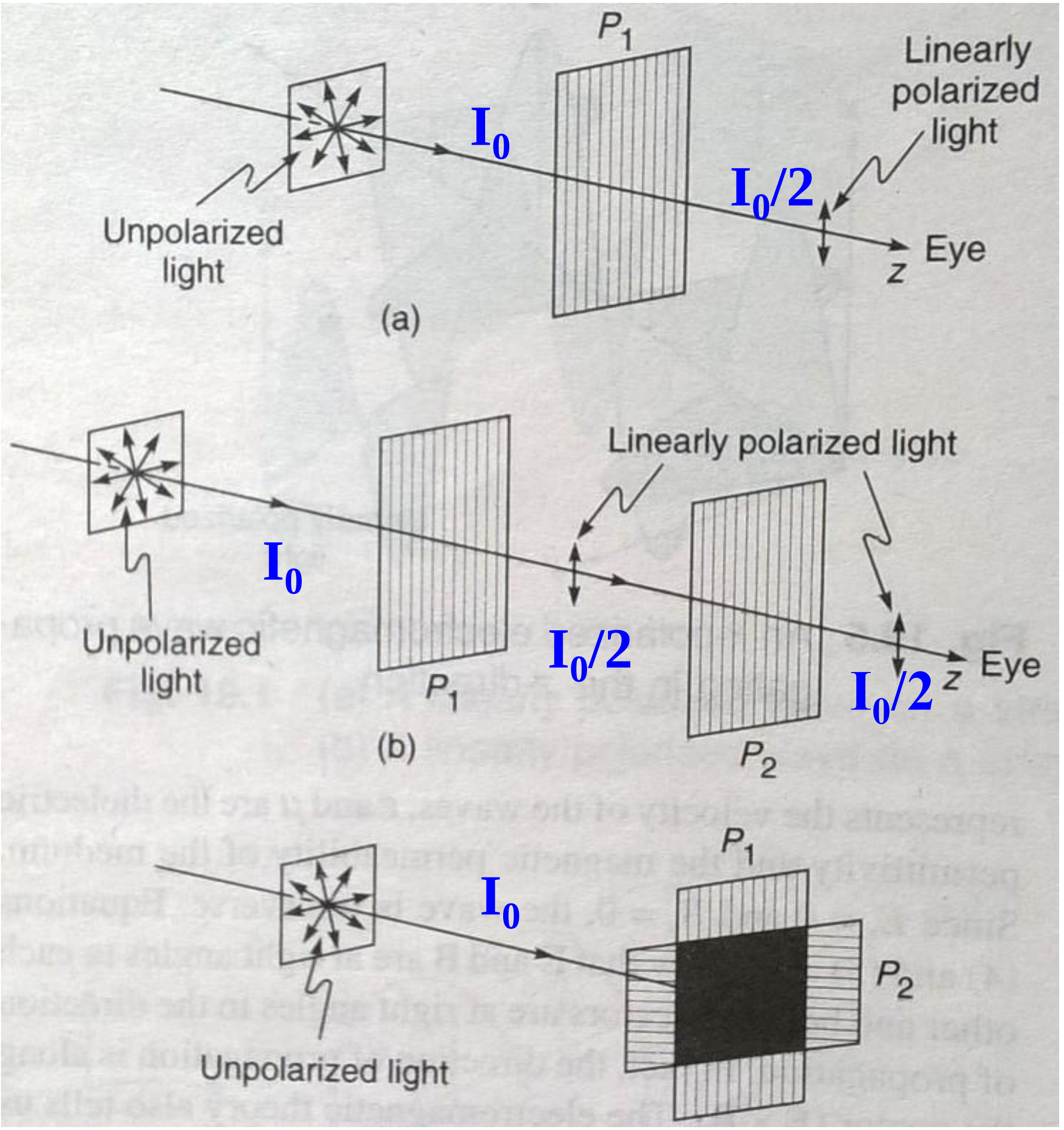
Linearly polarized EM wave



Ordinary EM wave - unpolarized

ELECTROMAGNETIC WAVE

- If an ordinary unpolarized light beam is allowed to fall on a polaroid P_1, the emerging beam will be linearly polarized and intensity will be halved.
- If it passes through a second polaroid P_2, the intensity of the emergent beam will depend on relative orientation of P_2 with respect to P_1.

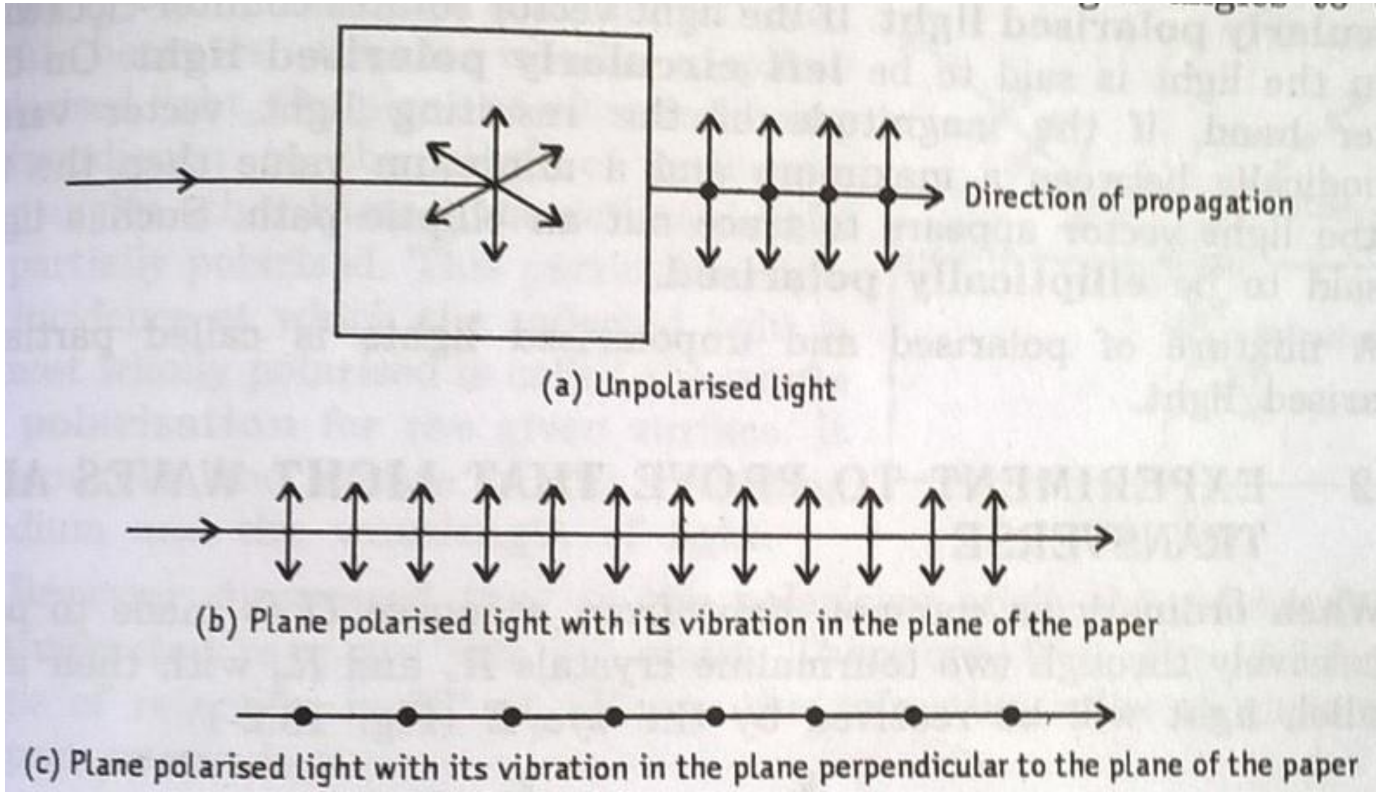


POLARIZATION OF EM WAVE

- Polarization is a property of the transverse wave that describes a regular orientation of vibration (for EM wave, the electric field vibration and magnetic field vibration) with respect to the direction of propagation.
- The direction of vibration may be oriented along a particular direction with respect to the propagation direction – **linearly/plane polarized light**.
- The plane containing the electric field vibration and the direction of propagation is called the **plane of vibration**. For a plane/linearly polarized light with its electric field vibration along x and propagating along z, x-z plane is the plane of vibration.

POLARIZATION OF EM WAVE

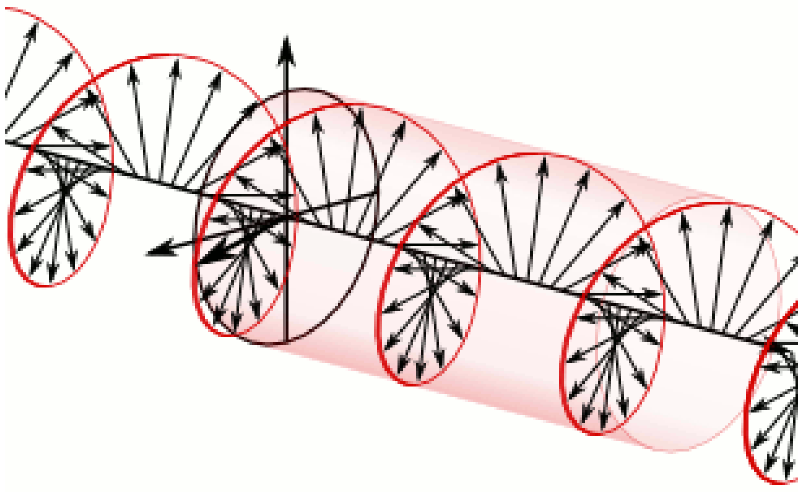
- The plane perpendicular to the plane of vibration and which does not contain the electric field vibration, is called the **plane of polarization**. For a plane/linearly polarized light with its electric field vibration along x and propagating along z, y-z plane is the plane of polarization.



Light: Ghosh & Mazumdar

POLARIZATION OF EM WAVE

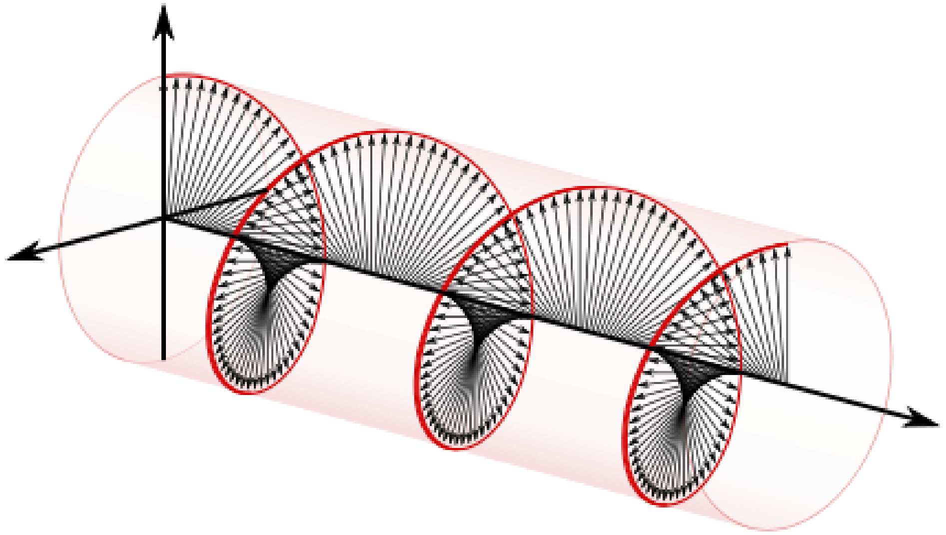
- By the superposition of two plane polarized wave under suitable conditions, the resultant field vector may be made to rotate in a plane perpendicular the direction of propagation.
- If the magnitude of the resulting field vector remains constant, the tip of the field vector appears to trace out a circle on a plane perpendicular to the propagation direction. – **Circularly polarized light**.
- If the magnitude of the electric field vector varies periodically between a maxima and a minima, the tip of the vector appears to trace an ellipse on a plane perpendicular to the propagation direction – **Elliptically Polarized Light**



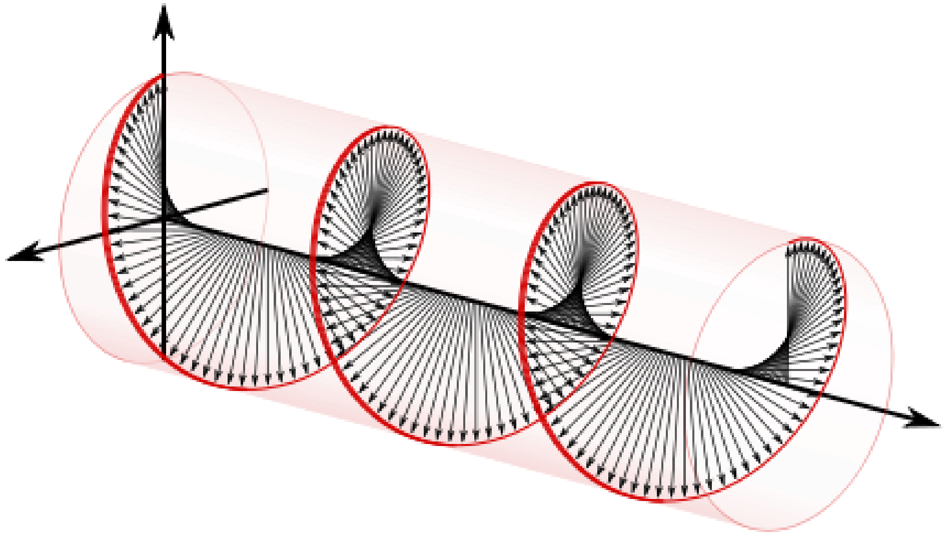
Wikipedia

POLARIZATION OF EM WAVE

- On looking towards the incoming light the resultant electric field vector appears to rotate clockwise – **Right Circularly/Elliptically Polarized Light**
- On looking towards the incoming light the resultant electric field vector appears to rotate anti-clockwise – **Left Circularly/Elliptically Polarized Light**



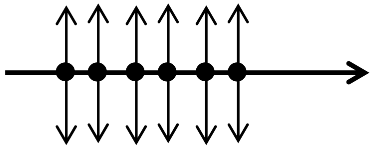
Left Circularly Polarized Light

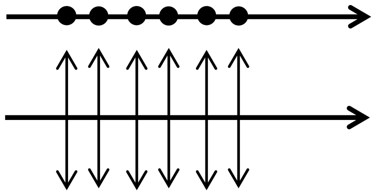


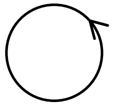
Right Circularly Polarized Light

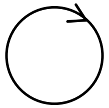
POLARIZATION

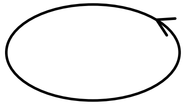
- EM Wave
  - Unpolarized

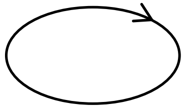
    
  - Polarized
    - Vibration along particular direction – Linear/Plane polarized

      
    - Vibration in rotating fashion
      - Circular polarization
        - Left

          
        - Right

          
      - Elliptical polarization
        - Left

          
        - Right

How to get polarized wave

1. Polarization by reflection

- If an ordinary (unpolarized) light is reflected at the interface of two media, the reflected and the transmitted rays both get partially polarized.
- For two particular media and a given wavelength of the incident EM wave, there exists a particular angle of incidence for which the reflected light is perfectly polarized with its electric field vector perpendicular to the plane of incidence – polarizing angle or, the Brewster's angle (θ_p).
- The polarizing angle of incidence is related to the relative refractive index as $\mu = \tan \theta_p$ – Brewster's law.

For air glass interface with $\mu = 1.5$, $\theta_p = 57°$

How to get polarized wave

- At the Brewster's angle of incidence, the reflected and the transmitted lights are mutually perpendicular.

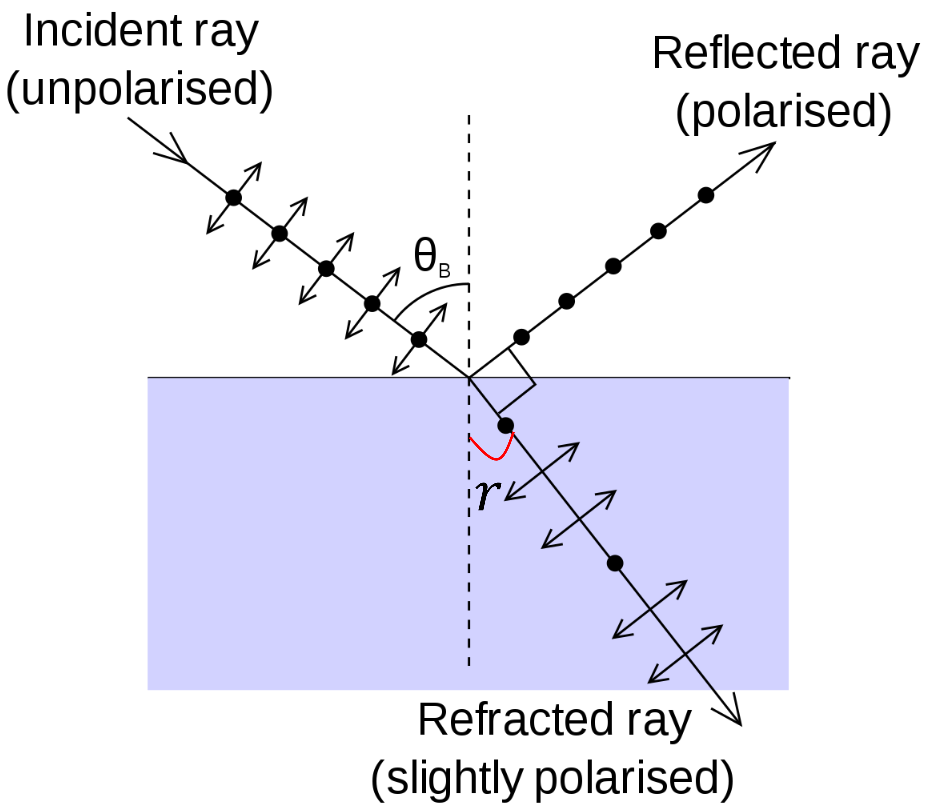
Brewster's law: $\mu = \tan\theta_p$

Snell's law: $\mu = \dfrac{\sin i}{\sin r} = \dfrac{\sin\theta_p}{\sin r}$

$$\therefore \frac{\sin\theta_p}{\sin r} = \tan\theta_p = \frac{\sin\theta_p}{\cos\theta_p}$$

$$\Rightarrow \sin r = \cos\theta_p = \sin(90° - \theta_p)$$

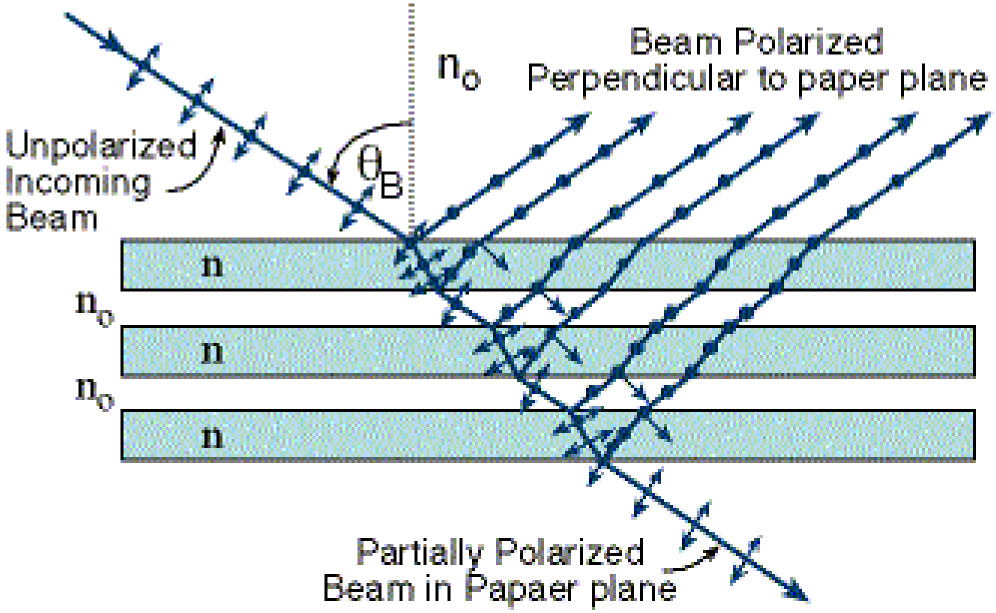
$$\boxed{\Rightarrow r + \theta_p = 90°}$$



How to get polarized wave

2. Polarization by refraction

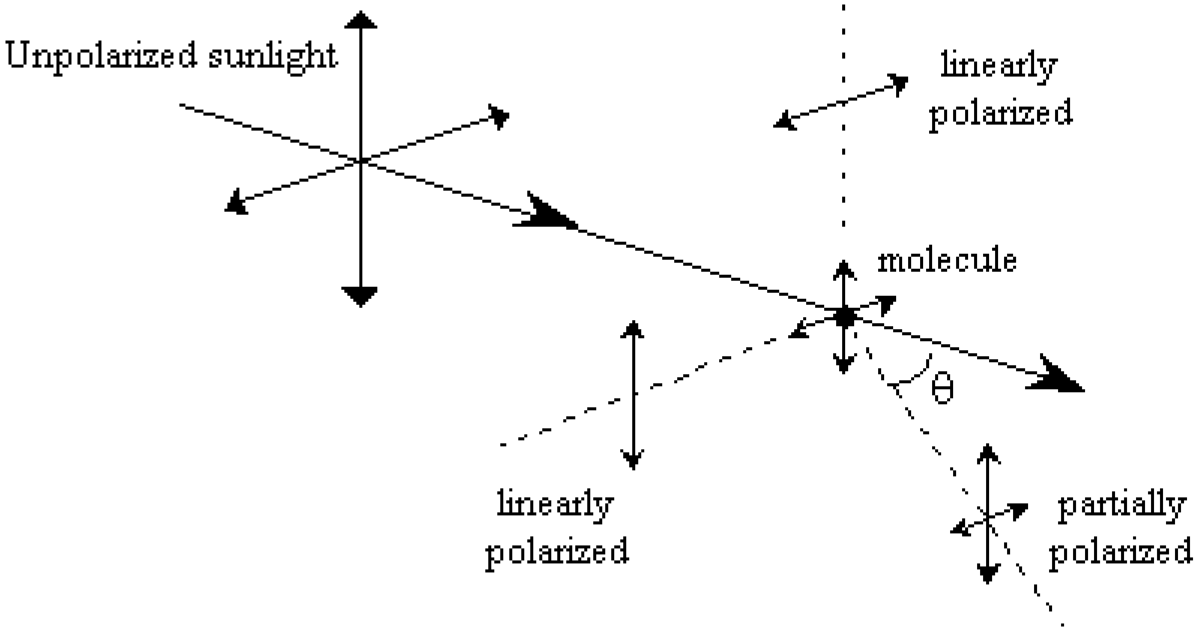
Successive refraction produces a linearly polarized light with its electric field vector in the plane of incidence.



How to get polarized wave

3. Polarization by scattering

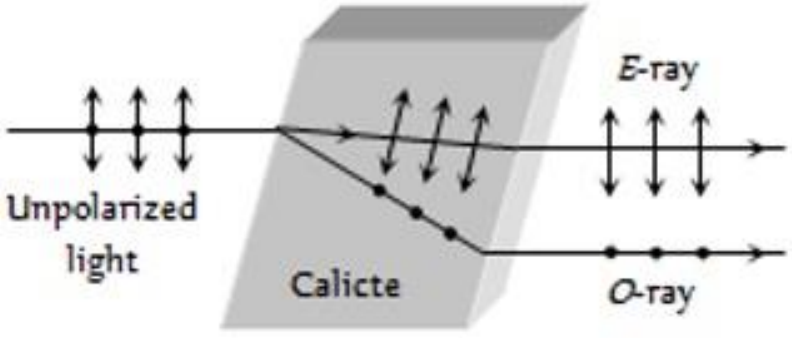
- If an unpolarized light is allowed to fall on a gas/liquid, the beam scattered at an angle 90° to the incident beam gets linearly polarized.
- The incident light induces dipole moment in the fluid molecules which in turn radiate polarized light.



How to get polarized wave

4. Polarization by double refraction

- There exists some crystals like calcite, tourmaline for which when an unpolarized light is incident it gets split into two rays.
- Both the rays are perfectly polarized in two mutually perpendicular directions.
- For the ordinary ray or O-ray, the electric field vector lies perpendicular to the plane of incidence while for the extraordinary ray or E-ray, the electric field lies on the plane of incidence.



How to get polarized wave

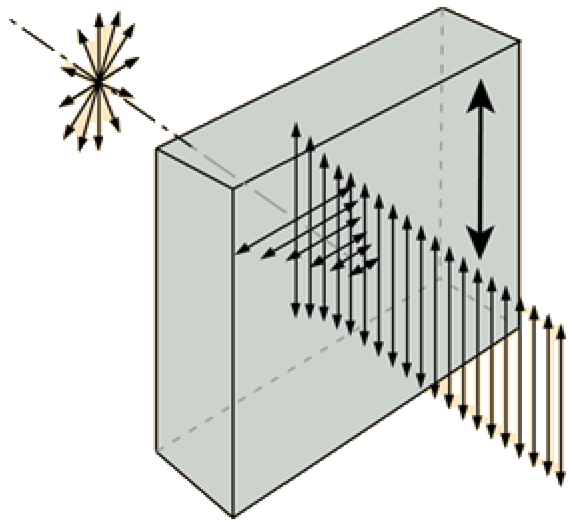
- The O-ray obeys the laws of refraction while the E-ray does not.
- The velocity of the rays within the crystals are different. Thus the crystal has two refractive indices for the O-ray and E-ray (at a given frequency of the incident light)

This phenomenon is known as **double refraction** or **birefringence**, and the crystals that exhibit such phenomenon are called doubly-refracting crystal or birefringent.

How to get polarized wave

5. Polarization by dichroism

- Some birefringent like tourmaline not only produce O-ray and E-ray but also absorb one of them much more strongly than the other.
- Thus, if an unpolarized light passes through such crystals of proper thickness, one of the linearly polarized components is completely absorbed and the other is transmitted in appreciable amount.
- This phenomenon is known as dichroism and the crystals like tourmaline which exhibit dichroism are called dichroic crystals.



How to get polarized wave

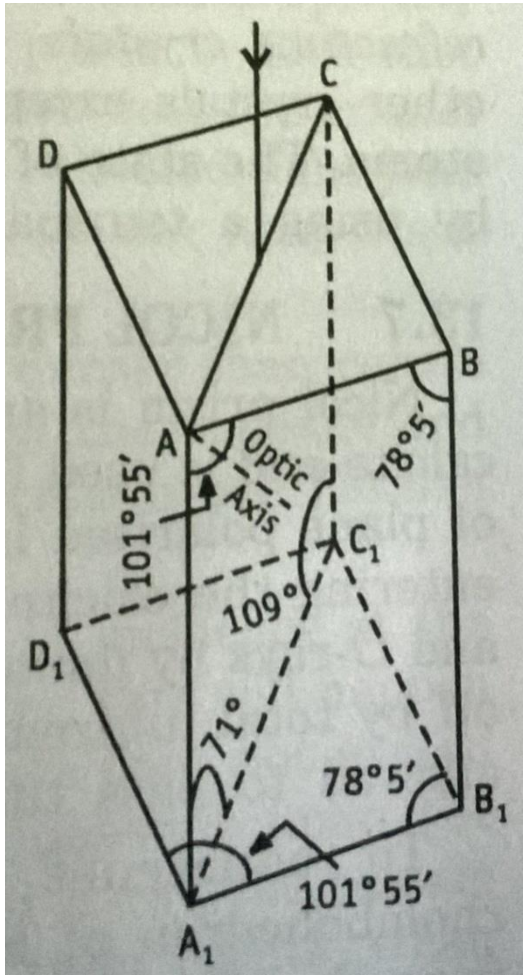
6. Polarization by polaroid

- Polaroid is a thin transparent film which can produce plane polarized light on transmission through it
- It usually consists of a thin film of nitrocellulose packed with tiny dichroic crystals (herapathite) with their optic axes all parallel
- Polaroids have wide applications in everyday life e.g., in sun glasses, wind screens, camera etc.

Optic Axis & Principal Section

Optic axis:

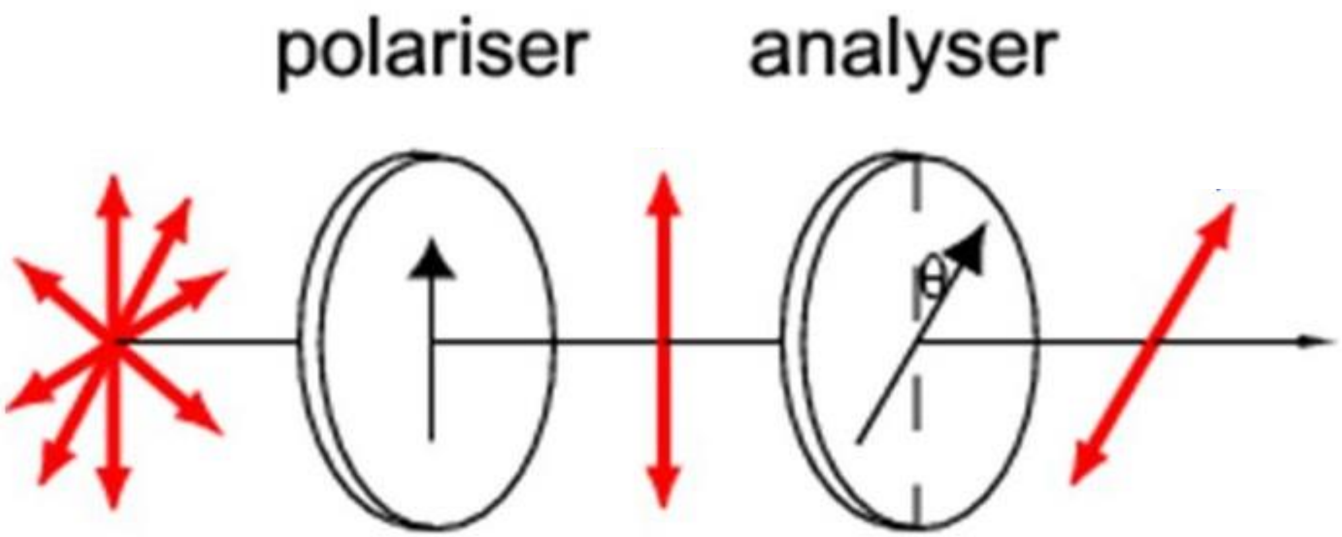
- Optic axis of a crystal is defined as the direction through it, along which if a ray travels then there will be no double refraction phenomenon and both the O-ray & E-ray travel with same velocity.
- A doubly refracting crystal possessing only one optic axis is called uniaxial crystal.



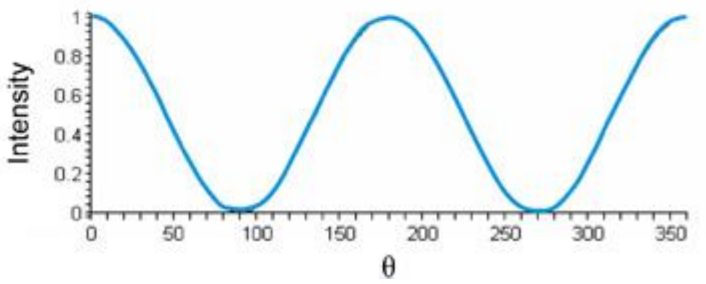
Principal Section:

- Principal section of a crystal is its section by a plane which passes through the optic axis of the crystal and is perpendicular to its two opposite refracting surfaces.

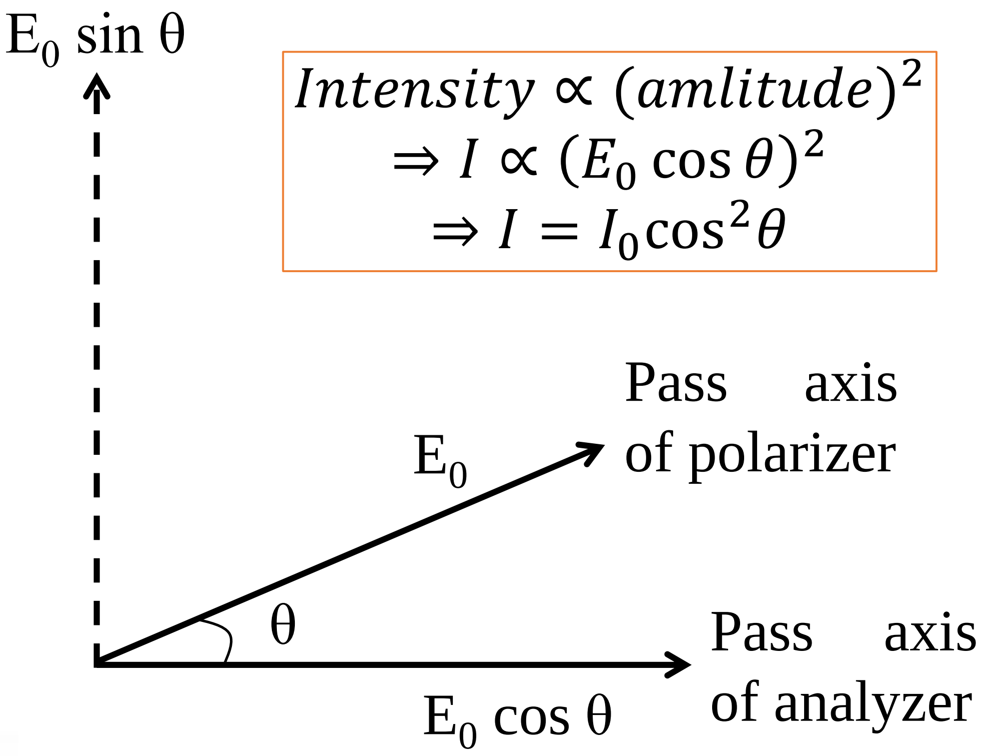
Law of Malus



$2I_0$ **I_0** **$I = ?$** $\boxed{I_0\cos^2\theta}$



$$\boxed{\begin{gathered} Intensity \propto (amlitude)^2 \\ \Rightarrow I \propto (E_0 \cos\theta)^2 \\ \Rightarrow I = I_0 \cos^2\theta \end{gathered}}$$

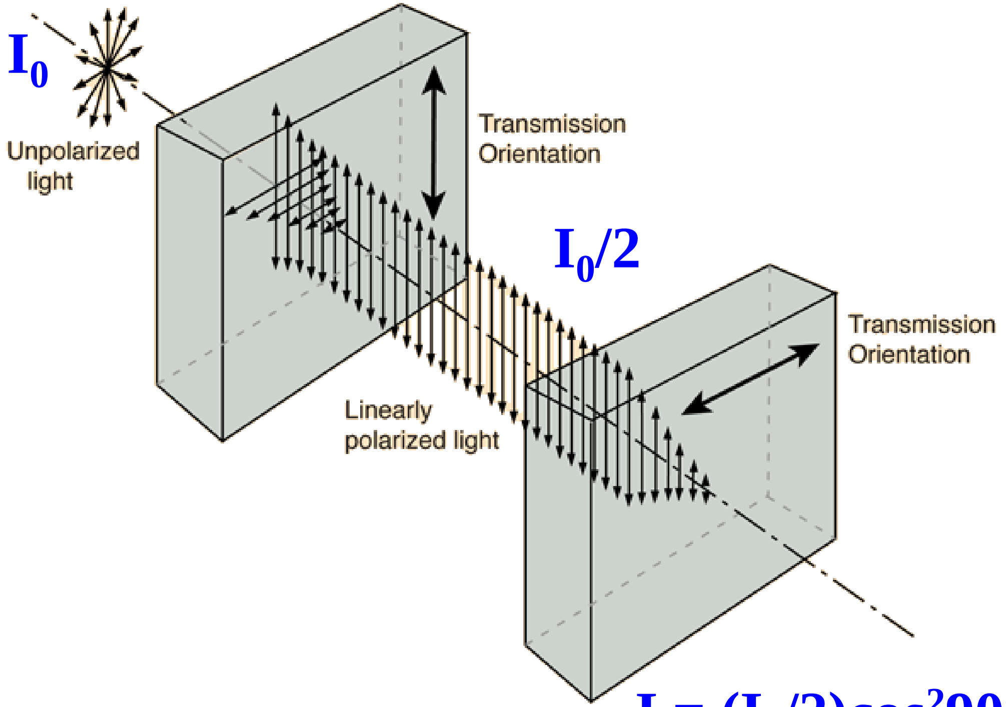


Law of Malus

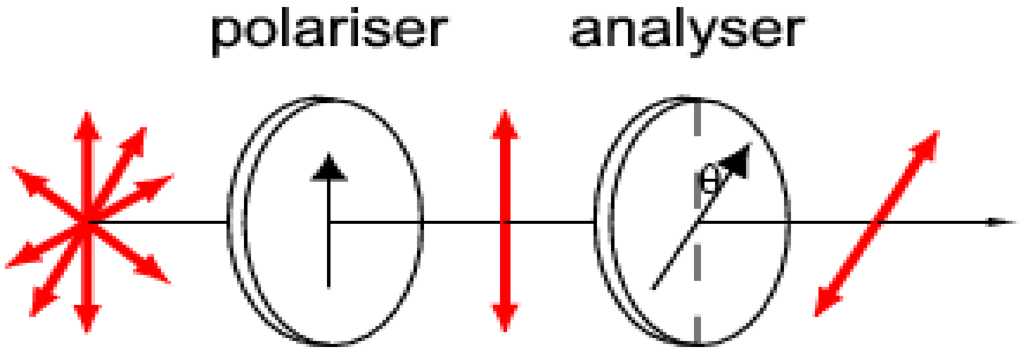
Prob. An unpolarized light of intensity I_0 is allowed to pass through two polaroid. What will be the intensity of the light after second polaroid if

(i) Optic axes of the polaroid are mutually perpendicular (cross polaroid)?

(ii) Optic axes are 45° with each other?



$I = (I_0/2)\cos^2 90° = 0$

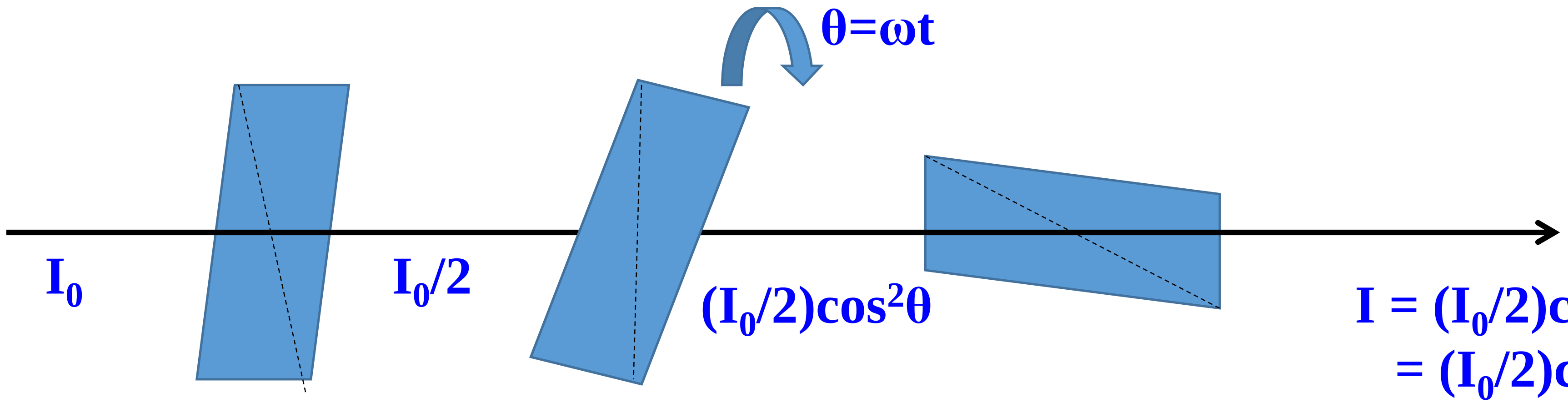


I_0 $\quad$ $I_0/2$

$I = (I_0/2)\cos^2 45°$
$= I_0/4$

Law of Malus

Prob. An unpolarized light of intensity I_0 is passed successively through 3 polaroids. The end polaroids have their pass axis perpendicular to each other while the polaroid at the middle rotated with angular frequency ω about the incident beam. Find the intensity of the emergent beam.



$$
\begin{aligned}
I &= (I_0/2)\cos^2\theta\cos^2(90°-\theta)\\
&= (I_0/2)\cos^2\theta\sin^2\theta\\
&= (I_0/8)\sin^2 2\theta\\
&= (I_0/8)\sin^2 2\omega t
\end{aligned}
$$

Superposition of Polarized Waves

1. Polarized along same direction:

$$\boxed{\begin{aligned}\vec{E}_1 &= \hat{e}E_{10}\cos(kz-\omega t)\\ \vec{E}_2 &= \hat{e}E_{20}\cos(kz-\omega t+\delta)\end{aligned}}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \hat{e}\{E_{10}\cos(kz-\omega t) + E_{20}\cos(kz-\omega t+\delta)\}$$

$$= \hat{e}[E_{10}\cos(kz-\omega t) + E_{20}\{\cos(kz-\omega t)\cos\delta - \sin(kz-\omega t)\sin\delta\}]$$

$$= \hat{e}[(E_{10} + E_{20}\cos\delta)\cos(kz-\omega t) - E_{20}\sin(kz-\omega t)\sin\delta\}]$$

$$= \hat{e}R\cos(kz-\omega t+\varphi)$$

$$R\cos\varphi = (E_{10} + E_{20}\cos\delta)$$

$$R\sin\varphi = E_{20}\sin\delta$$

Linearly Polarized

Superposition of Polarized Waves

2. Polarized along perpendicular direction:

$$\vec{E}_1 = \hat{x}E_{10}\cos(kz - \omega t)$$

$$\vec{E}_2 = \hat{y}E_{20}\cos(kz - \omega t + \delta)$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \hat{x}E_{10}\cos(kz - \omega t) + \hat{y}E_{20}\cos(kz - \omega t + \delta)\}$$

At z = 0 plane:

$$E_1 = E_{10}\cos\omega t \quad \dots\dots\dots (1)$$

$$E_2 = E_{20}\cos(\omega t - \delta) \quad \dots\dots .. (2)$$

From eq. (2):

$$E_2 = E_{20}(\cos\omega t\cos\delta - \sin\omega t\sin\delta)$$

$$= E_{20}\left\{\frac{E_1}{E_{10}}\cos\delta - \sqrt{1 - \left(\frac{E_1}{E_{10}}\right)^2}\sin\delta\right\}$$

From eq. (1):

$$\frac{E_1}{E_{10}} = \cos\omega t$$

Superposition of Polarized Waves

$$\Rightarrow \sqrt{1-\left(\frac{E_1}{E_{10}}\right)^2}\sin\delta = \left(\frac{E_2}{E_{20}} - \frac{E_1}{E_{10}}\cos\delta\right)$$

$$\Rightarrow \left\{1-\left(\frac{E_1}{E_{10}}\right)^2\right\}\sin^2\delta = \left(\frac{E_2}{E_{20}}\right)^2 + \left(\frac{E_1}{E_{10}}\right)^2\cos^2\delta - 2\frac{E_1}{E_{10}}\frac{E_2}{E_{20}}\cos\delta$$

$$\boxed{\Rightarrow \frac{E_1^2}{E_{10}^2} + \frac{E_2^2}{E_{20}^2} - 2\frac{E_1E_2}{E_{10}E_{20}}\cos\delta = \sin^2\delta \quad \dots\dots(3)}$$

Nature of polarization of the resultant wave is determined by δ, E_{10}, E_{20}.

Superposition of Polarized Waves

$$\frac{E_1^2}{E_{10}^2} + \frac{E_2^2}{E_{20}^2} - 2\frac{E_1 E_2}{E_{10}E_{20}}\cos\delta = \sin^2\delta$$

Special Cases:

I $\boxed{\delta = 2n\pi, n = 0, 1, 2, \ldots.}$

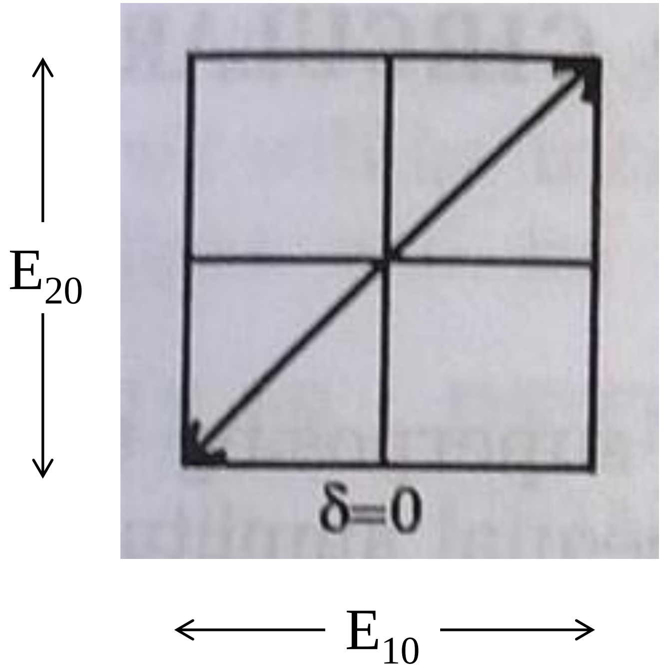
$$E_1 = E_{10}\cos\omega t \quad \ldots\ldots\ldots (1)$$

$$E_2 = E_{20}\cos\omega t \quad \ldots\ldots.. (2)$$

$$\frac{E_1^2}{E_{10}^2} + \frac{E_2^2}{E_{20}^2} - 2\frac{E_1 E_2}{E_{10}E_{20}} = 0$$

$$\boxed{\Rightarrow E_2 = \frac{E_{20}}{E_{10}} E_1}$$

Linearly Polarized



Superposition of Polarized Waves

$$\frac{E_1^2}{E_{10}^2}+\frac{E_2^2}{E_{20}^2}-2\frac{E_1E_2}{E_{10}E_{20}}\cos\delta=\sin^2\delta$$

Special Cases:

II $\boxed{\delta=(2n+1)\pi, n=0,1,2,\ldots.}$

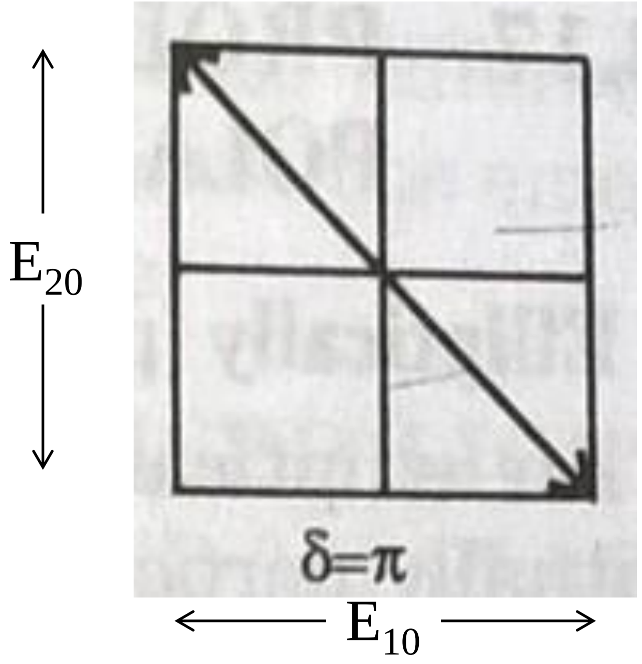
$$E_1=E_{10}\cos\omega t \quad \ldots\ldots\ldots(1)$$

$$E_2=-E_{20}\cos\omega t \quad \ldots\ldots..(2)$$

$$\frac{E_1^2}{E_{10}^2}+\frac{E_2^2}{E_{20}^2}+2\frac{E_1E_2}{E_{10}E_{20}}=0$$

$$\boxed{\Rightarrow E_2=-\frac{E_{20}}{E_{10}}E_1}$$

Linearly Polarized



Superposition of Polarized Waves

$$\frac{E_1^2}{E_{10}^2}+\frac{E_2^2}{E_{20}^2}-2\frac{E_1E_2}{E_{10}E_{20}}\cos\delta=\sin^2\delta$$

Special Cases:

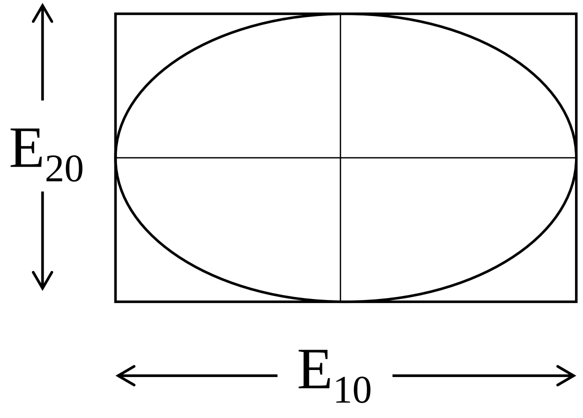
III $\boxed{\delta=(2n+1)\pi/2, n=0,1,2,\ldots.}$

$$E_1=E_{10}\cos\omega t \quad \ldots\ldots\ldots(1)$$

$$E_2=\pm E_{20}\sin\omega t \ldots\ldots\ldots(2)$$

$$\boxed{\frac{E_1^2}{E_{10}^2}+\frac{E_2^2}{E_{20}^2}=1}$$

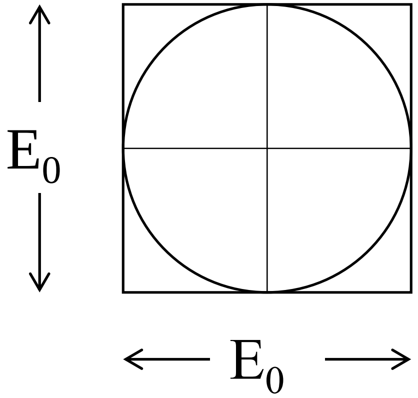
Elliptically Polarized



If $E_{10}=E_{20}=E_0$ (say),

$$\boxed{E_1^2+E_2^2=E_0^2}$$

Circularly Polarized



Superposition of Polarized Waves

Special Cases:

$$\frac{E_1^2}{E_{10}^2} + \frac{E_2^2}{E_{20}^2} - 2\frac{E_1 E_2}{E_{10}E_{20}}\cos\delta = \sin^2\delta$$

III (a) $\boxed{\delta = (2n+1)\pi/2, n = 0, 2, 4 \ldots.}$

$$E_1 = E_{10}\cos\omega t \; \ldots\ldots\ldots (1)$$

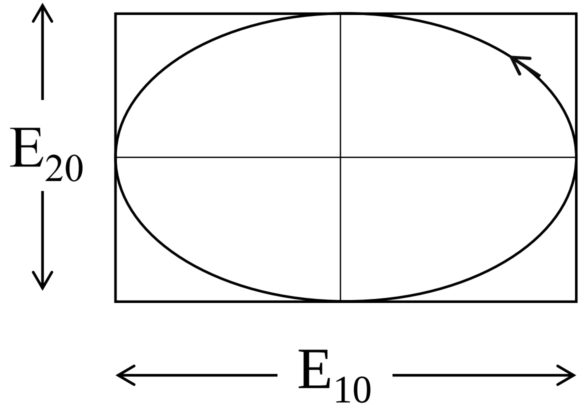
$$E_2 = E_{20}\sin\omega t \ldots\ldots.. (2)$$

$$t = 0: \qquad E_1 = E_{10}, E_2 = 0$$

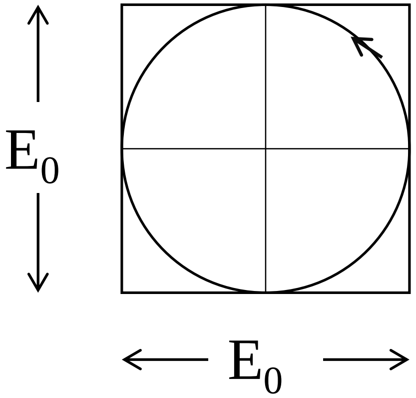
$$t = \frac{\pi}{2\omega}: \qquad E_1 = 0, E_2 = E_{20}$$

$$t = \frac{\pi}{\omega}: \qquad E_1 = -E_{10}, E_2 = 0$$

$$t = \frac{3\pi}{2\omega}: \qquad E_1 = 0, E_2 = -E_{20}$$



Left-Elliptically Polarized



Left-Circularly Polarized

Superposition of Polarized Waves

Special Cases:

III (b) $\boxed{\delta = (2n+1)\pi/2, n = 1, 3, 5 \ldots.}$

$$E_1 = E_{10}\cos\omega t \;\ldots\ldots\ldots(1)$$

$$E_2 = -E_{20}\sin\omega t \ldots\ldots..(2)$$

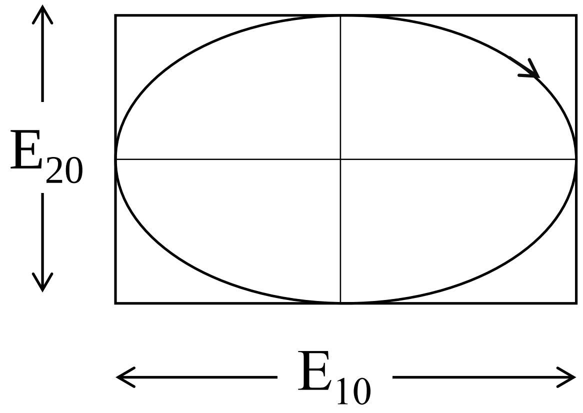
$$t = 0: \qquad E_1 = E_{10}, E_2 = 0$$

$$t = \frac{\pi}{2\omega}: \qquad E_1 = 0, E_2 = -E_{20}$$

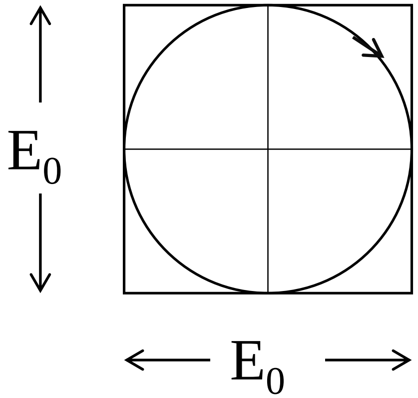
$$t = \frac{\pi}{\omega}: \qquad E_1 = -E_{10}, E_2 = 0$$

$$t = \frac{3\pi}{2\omega}: \qquad E_1 = 0, E_2 = E_{20}$$

$$\frac{E_1^2}{E_{10}^2} + \frac{E_2^2}{E_{20}^2} - 2\frac{E_1E_2}{E_{10}E_{20}}\cos\delta = \sin^2\delta$$



Right-Elliptically Polarized



Right-Circularly Polarized

Superposition of Polarized Waves

Problem: Describe the state of polarization of the EM wave represented by

$$E_x = E_0 \cos(kz - \omega t)\,, E_y = E_0 \sin(kz - \omega t)$$

$$\boxed{E_x^2 + E_y^2 = E_0^2} \longleftarrow \text{Circularly Polarized}$$

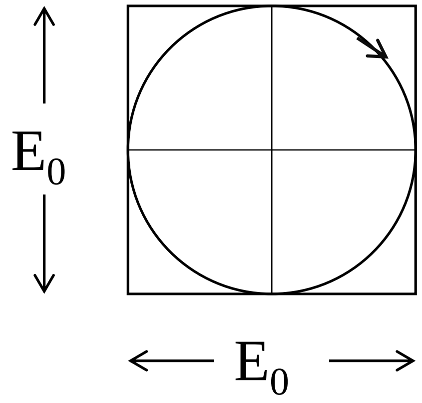
Consider $z = 0$ plane: $E_x = E_0 \cos \omega t\,, E_y = -E_0 \sin \omega t$

$t = 0$: $\quad E_x = E_0, E_y = 0$

$t = \dfrac{\pi}{2\omega}$: $\quad E_x = 0, E_y = -E_0$

$t = \dfrac{\pi}{\omega}$: $\quad E_x = -E_0, E_y = 0$

$t = \dfrac{3\pi}{2\omega}$: $\quad E_x = 0, E_y = E_0$



Right-Circularly Polarized

Retardation Plates

- A plate made of doubly refracting crystal with its refracting faces cut parallel to the optic axis.
- As the ray enters the plate, it splits into O-ray and E-ray
- The O-ray and E-ray travels with different speed through the crystal. In other words, the crystal has different refractive indices for the O-ray and E-ray
- Hence a path difference or phase difference is introduced between O-ray and E-ray – that justifies the terminology ‘retardation plate’
- If μ_o and μ_e are the refractive indices of the crystal for a wave of wavelength λ, the optical path difference introduced after travelling a distance *d* through the plate is

$$\Delta = (\mu_0 \sim \mu_e)d$$

Corresponding phase difference $$\delta = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda}(\mu_0 \sim \mu_e)d$$

Retardation Plates

- **Quarter-Wave Plate (λ/4 plate):** Thickness of the plate is such that for a given wavelength, it introduces a path difference λ/4 (phase difference π/2)

$$\Delta = (\mu_0 \sim \mu_e)d = \frac{\lambda}{4}$$
$$\Rightarrow d = \frac{\lambda}{4(\mu_0 \sim \mu_e)}$$

- **Half-Wave Plate (λ/2 plate):** Thickness of the plate is such that for a given wavelength, it introduces a path difference λ/2 (phase difference π)

$$\Delta = (\mu_0 \sim \mu_e)d = \frac{\lambda}{2}$$
$$\Rightarrow d = \frac{\lambda}{2(\mu_0 \sim \mu_e)}$$

Retardation Plates

Problem: Compute the minimum thickness of a quartz retarder must have to be a quarter-wave plate for incident light of wavelength 6000 Å. What thickness is required for half-wave plate for same wavelength? Given, $n_e = 1.4864, n_o = 1.658$

For quarter-wave plate: $$d_q = \frac{\lambda}{4(\mu_0 \sim \mu_e)} = \frac{6000 \times 10^{-10}}{4(1.658 - 1.4864)} \text{ m} = 0.874 \text{ µm}$$

For half-wave plate: $$d_h = \frac{\lambda}{2(\mu_0 \sim \mu_e)} = \frac{6000 \times 10^{-10}}{2(1.658 - 1.4864)} \text{ m} = 1.75 \text{ µm}$$

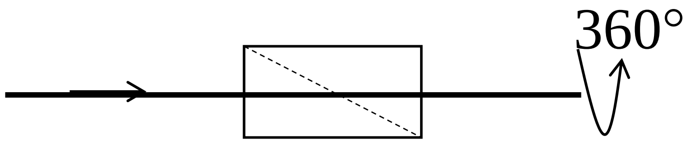
Production & Detection of Polarized Waves

Summary so far:

- A linear/plane polarized wave has its electric field vibration oriented along a particular direction with respect to the propagation direction.
- A circularly polarized wave is a superposition of two linear/plane polarized waves with their electric field vibrations of same amplitude and in perpendicular direction.
- A elliptically polarized wave is a superposition of two linear/plane polarized waves with their electric field vibrations of different amplitude and in perpendicular direction.
- An unpolarized waves on passing through a polarizer becomes linear/plane polarized.
- A quarter-wave plate creates a phase difference of $\pi/2$ between two perpendicularly polarized waves.
- A half-wave plate creates a phase difference of π between two perpendicularly polarized waves.

Production & Detection of Polarized Waves

Summary so far:



Experiment: Rotate a polaroid through 360° about the incident beam

Observation	Conclusion
Intensity becomes halved but does not vary while rotating the polaroid	The incident beam is unpolarized or circularly polarized or a mixture of them
Intensity varies periodically with two completely extinguished minima and two maxima	The incident beam is linear/plane polarized
Intensity varies periodically with two minima and two maxima, however minima is not completely extinguished	The incident beam is elliptically polarized or a mixture of elliptically polarized and unpolarized, or a mixture of linearly polarized and unpolarized

Production & Detection of Polarized Waves

1. Unpolarized Light

Production: Ordinary lights are unpolarized!

Detection: Pass the original light through a polaroid and rotate it through 360° about the incident beam

Observation: Intensity becomes halved but it does not vary while rotating the polaroid

2. Linear/Plane Polarized Light

Production: Pass the ordinary unpolarized light through a polaroid

Detection: Pass the linear/plane polarized light through a second polaroid and rotate it through 360° about the incident beam

Observation: Intensity varies periodically with two completely extinguished minima and two maxima

Production & Detection of Polarized Waves

3. Elliptically Polarized Light

Production:

- Plane polarized light is incident normally on a quarter-wave plate with the electric field vibration making an angle θ other than 45° with the optic axis of the plate.
- It splits into two polarized components with their electric field amplitudes unequal and direction of vibration mutually perpendicular.
- On passing through the $\lambda/4$ plate a relative phase difference of $\pi/2$ is introduced between them and the emergent beam is elliptically polarized.

Production & Detection of Polarized Waves

3. Elliptically Polarized Light

Detection:

Step 1: Pass the emergent wave through a polaroid & rotate it through 360° about the incident beam

Observation: varies periodically with two minima and two maxima – minima however not completely extinguished

Step 2: Allow original wave to pass through $\lambda/4$ plate followed by a rotating polaroid

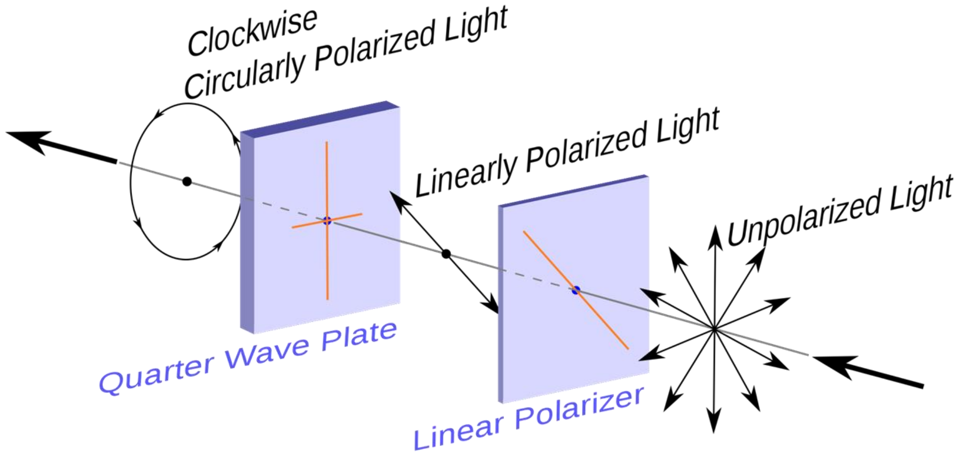
Observation: Intensity varies periodically with two completely extinguished minima and two maxima

Production & Detection of Polarized Waves

4. Circularly Polarized Light

Production:

- Plane polarized light is incident normally on a quarter-wave plate with the electric field vibration making an angle $\theta = 45°$ with the optic axis of the plate.
- It splits into two polarized components with their electric field amplitudes unequal and direction of vibration mutually perpendicular.
- On passing through the $\lambda/4$ plate a relative phase difference of $\pi/2$ is introduced between them and the emergent beam is elliptically polarized.



Production & Detection of Polarized Waves

4. Circularly Polarized Light

Detection:

Step 1: Pass the emergent wave through a polaroid & rotate it through 360° about the incident beam

Observation 1: Intensity does not vary

Step 2: Allow original wave to pass through $\lambda/4$ plate followed by a rotating polaroid

Observation 2: Intensity varies periodically with two completely extinguished minima and two maxima

Production & Detection of Polarized Waves

Incident EM Wave

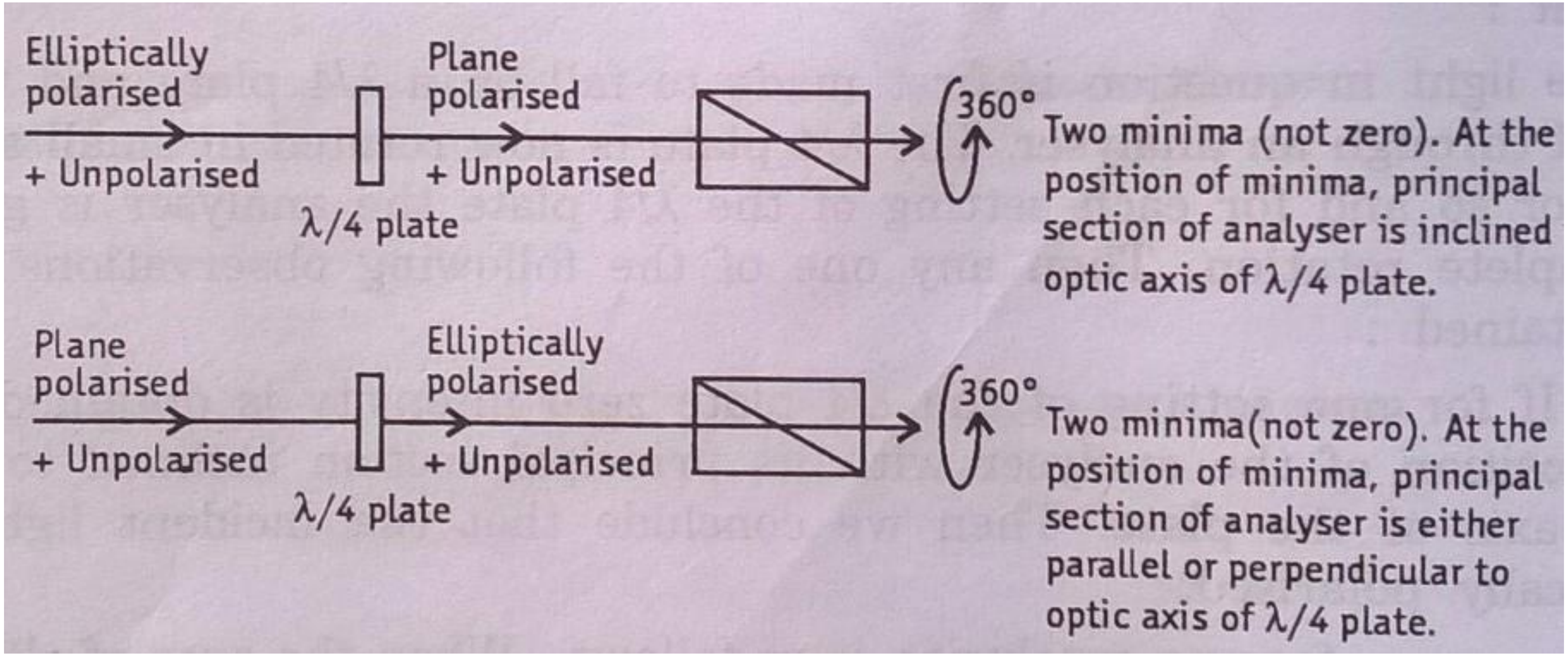
↓

Rotating Polaroid

- **Observation 1**: unpolarized / circularly polarized / (unpolarized + circularly polarized)
  - Allow original wave to pass through $\lambda/4$ plate followed by a rotating polaroid
    - **Observation 1**: unpolarized
    - **Observation 2**: circularly polarized
    - **Observation 3**: unpolarized + circularly polarized
- **Observation 2**: linear / plane polarized
- **Observation 3**: elliptic / (plane polarized + unpolarized) / (elliptic + unpolarized)
  - Allow original wave to pass through $\lambda/4$ plate followed by a rotating polaroid
    - **Observation 2**: elliptically polarized
    - **Observation 3**: (plane polarized + unpolarized) / (elliptic + unpolarized)

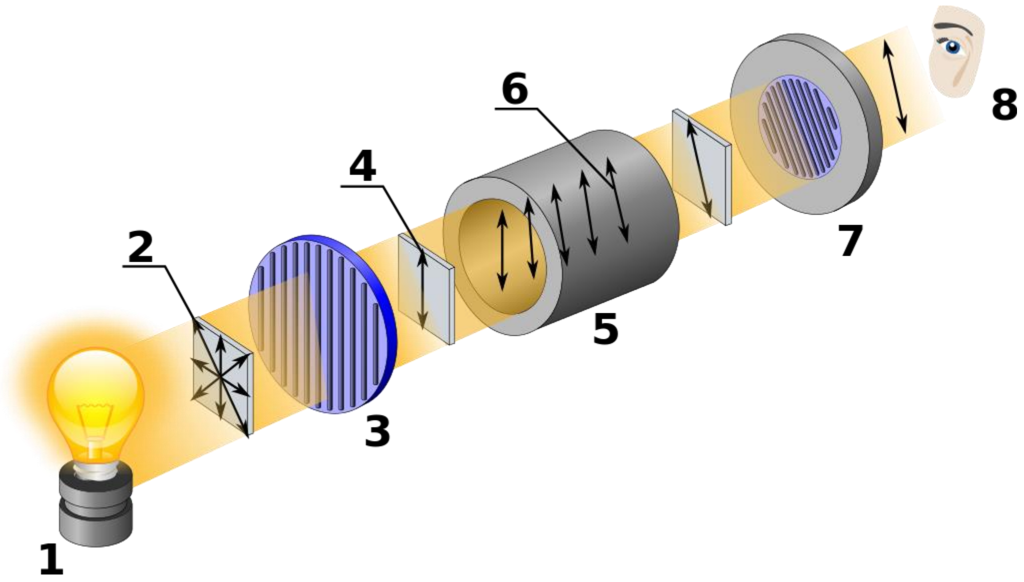
Production & Detection of Polarized Waves

How to distinguish between a mixture of plane polarized and unpolarized light, and a mixture of elliptically polarized and unpolarized light?



Optical Activity

- Optical rotation or optical activity (sometimes referred to as rotary polarization) is the rotation of the plane of polarization of linearly polarized light as it travels through certain materials.
- Optical activity occurs only in chiral materials, those lacking microscopic mirror symmetry.
- Unlike other sources of birefringence, optical activity can be observed in fluids. This can include gases or solutions of chiral molecules such as sugars, molecules with helical secondary structure such as some proteins, and also chiral liquid crystals. It can also be observed in chiral solids such as certain crystals with a rotation between adjacent crystal planes (such as quartz) or metamaterials.



Wikipedia

Optical Activity

- The rotation of the plane of polarization may be either clockwise, to the right (dextrorotary — d-rotary), or left (levorotary — l-rotary) depending on which stereoisomer is present (or dominant). For instance, sucrose and camphor are d-rotary whereas cholesterol is l-rotary.
- For a given substance, at a given temperature, the angle by which the polarization of light of a specified wavelength is rotated is proportional to the path length (*l*) through the material and (for a solution) proportional to its concentration (*c*). The rotation is not dependent on the direction of propagation.

$$\boxed{\theta = slc}$$

s = specific rotation

Wikipedia

Optical Activity

- The amount of rotation varies with temperature of the solution and the wavelength of the incident light. The angle of rotation is found to vary as inverse square of the wavelength. Thus if a plane polarized polychromatic light is passed through such optically active substance, it will decompose into constituent clours – a phenomenon known as rotary dispersion.
- The combined rotation produced by a number of optically active substances is the algebraic sum of the rotation caused by each separately.

Two solutions produce 34° right handed and 24° left handed rotations respectively in an experiment. What amount of rotation would be produced by an equal mixture (volume) of the two solutions?

$$\theta = \theta_1 - \theta_2 = \frac{1}{2}(34° - 24°) = 5° \qquad \text{right handed rotation}$$

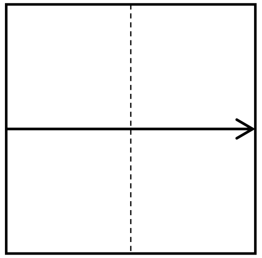
Wikipedia

Fresnel's Theory

- A plane polarized light falling on an optically active medium along its optic axis splits up into two circularly polarized vibrations of equal amplitudes and rotating in opposite directions –one clockwise and other anticlockwise.
- In an **optically active crystal**, like quartz , two circular components travel with different speeds so that relative phase difference is developed between them. In dextro-rotatory substance $v_r > v_l$ and in leavo rotatory substance $v_l > v_r$.
- On emergence from an optically active substance the two circular vibrations recombine to give plane polarized light whose plane of vibration has been rotated w.r.t. that of incident light through a certain angle depends on the phase diff between the two vibrations.

Wikipedia

Fresnel's Theory



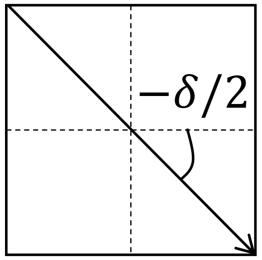
Incident linearly polarized light: $x = 2a\cos\omega t$

Left circularly polarized $\begin{cases} x_1 = a\cos\omega t \\ y_1 = a\sin\omega t \end{cases}$ $\quad$ $\begin{cases} x_2 = a\cos\omega t \\ y_2 = -a\sin\omega t \end{cases}$ Right circularly polarized

Emergent light: $\begin{cases} x_1 = a\cos\omega t \\ y_1 = a\sin\omega t \end{cases}$ $\quad$ $\begin{cases} x_2 = a\cos(\omega t+\delta) \\ y_2 = -a\sin(\omega t+\delta) \end{cases}$

$$x = x_1 + x_2 = a\cos\omega t + a\cos(\omega t+\delta) = 2a\cos\frac{\delta}{2}\cos\left(\omega t+\frac{\delta}{2}\right)$$

$$y = y_1 + y_2 = a\sin\omega t - a\sin(\omega t+\delta) = -2a\sin\frac{\delta}{2}\cos\left(\omega t+\frac{\delta}{2}\right)$$



$$\boxed{y = \tan\left(-\frac{\delta}{2}\right)x}$$

Fresnel's Theory

- v_l and v_r are respectively the velocities of the left and right circularly polarized light within the optically active substance, the time difference to travel the substance $t = l/v_l \sim l/v_r$
- The phase difference introduced between the two circular vibrations is

$$\delta = \frac{2\pi}{T} t = \frac{2\pi}{T}\left(\frac{l}{v_l} \sim \frac{l}{v_r}\right)$$

$$\text{or, } \delta = \frac{2\pi l}{cT}\left(\frac{c}{v_l} \sim \frac{c}{v_r}\right) = \frac{2\pi l}{\lambda}(n_l \sim n_r)$$

- The rotation of the plane of vibration is

$$\boxed{\phi = \frac{\delta}{2} = \frac{\pi l}{\lambda}(n_l \sim n_r)}$$

Fresnel's Theory

- λ_l and λ_r are respectively the wavelengths of the left and right circularly polarized light within the optically active substance ($\lambda_l = v_l T$, $\lambda_r = v_r T$), the phase difference introduced after they travel the substance

$$\delta = 2\pi l\left(\frac{1}{\lambda_l} \sim \frac{1}{\lambda_r}\right)$$

$$\text{or}, \delta = 2\pi l\frac{\Delta\lambda}{\lambda^2} \qquad [\lambda_l = \lambda, \lambda_r = \lambda + \Delta\lambda]$$

- The rotation of the plane of vibration is

$$\boxed{\phi = \frac{\delta}{2} = \pi l\frac{\Delta\lambda}{\lambda^2}}$$

Fresnel's Theory

Calculate the difference between the refractive indices for left and right circularly polarized light (λ = 600 nm) for a quartz plate (specific rotation 25 deg/mm). [CU – 2010]

$$\theta = \frac{\phi}{l} = \frac{\pi}{\lambda}(n_l \sim n_r)$$

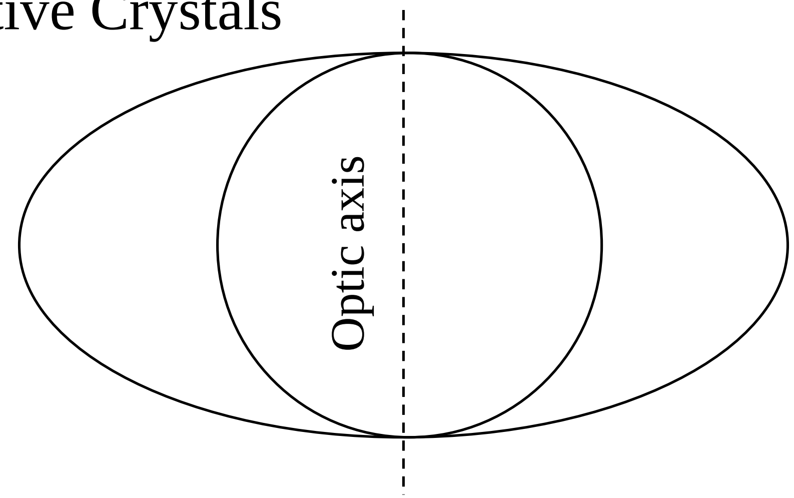
$$n_l \sim n_r = \frac{\theta\lambda}{\pi} = \frac{25 \times 600 \times 10^{-9}}{180 \times 0.001 \times 3.14} = 0.0000265$$

Huygen's Theory

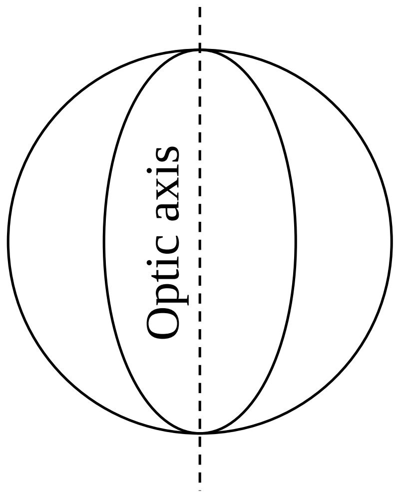
- Every point on a doubly refracting uniaxial crystal disturbed by the incident light becomes the source of secondary wavelets – spherical for O-ray and ellipsoid of revolution about the optic axis for E-ray.
- O-ray travels with same velocity in all directions while the E-ray has different velocity in different directions.
- The crystal has different refractive indices for the O-ray and E-ray ($\mu_o \neq \mu_e$)
- The optical properties of the uniaxial crystals are perfectly symmetrical about the optic axis – O-ray and E-ray has the same velocity along the optic axis.

Huygen's Theory

- In some crystals like calcite, O-ray travels with smaller velocity than E-ray (i.e. $\mu_o > \mu_e$) in the direction normal to the optic axis – the spherical wavelet is contained within the ellipsoid of revolution: Negative Crystals
- In some crystals like quartz, O-ray travels with greater velocity than E-ray (i.e. $\mu_o < \mu_e$) in the direction normal to the optic axis – the ellipsoid of revolution is contained within the spherical wavelet: Positive Crystals



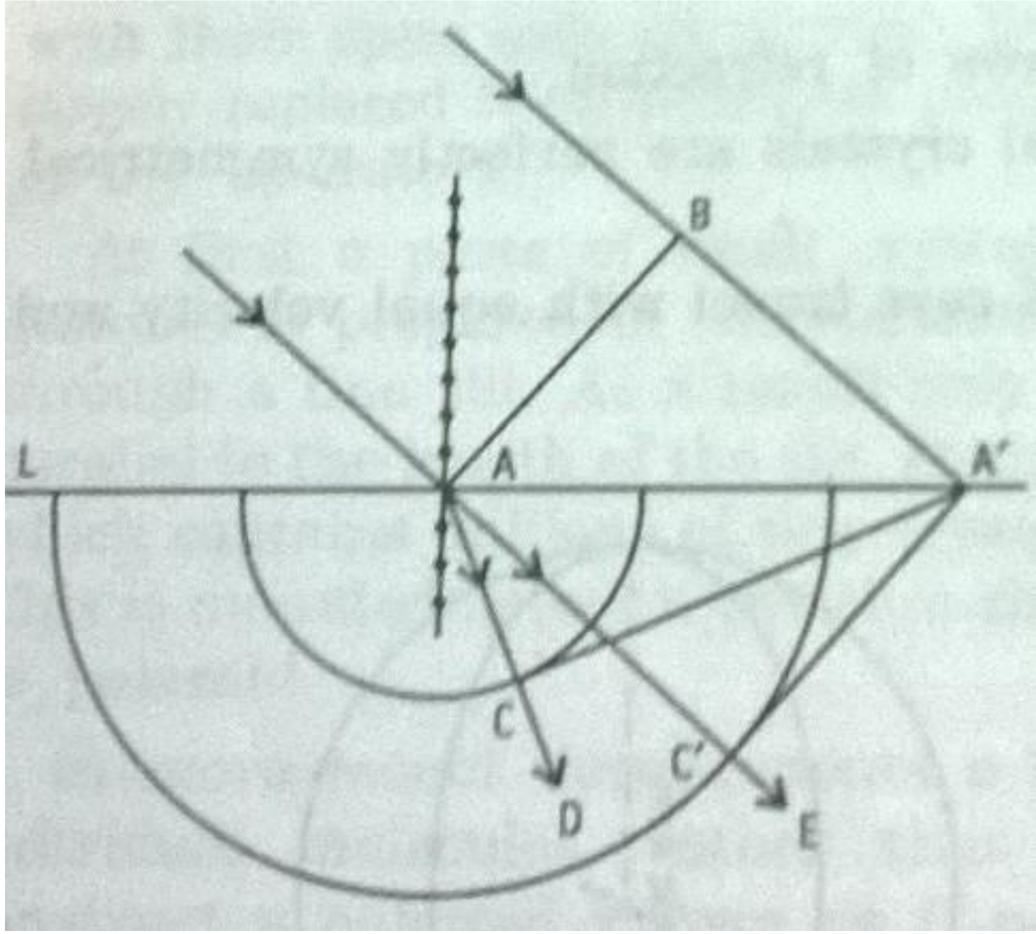
Negative Crystal



Positive Crystal

Huygen's Theory

Construction of wave surface on the plane of incidence: Optic axis perpendicular to the plane of incidence and parallel to the crystal surface



$$\frac{BA'}{v} = \frac{AC}{v_0} = \frac{AC'}{v_e}$$

Light: Ghosh & Mazumdar