FOUCAULT'S PENDULUM

We should start our discussion by recapitulating the concept of rotating frames & inertial frames (i.e. the frame w.r.t. which rotation is considered). If the frame rotates with angular velocity $\vec{\omega}$ then $\left.\frac{d}{dt}\right|_{inertial} = \left.\frac{d}{dt}\right|_{rotating} + \vec{\omega} \times$

If we symbolise $\frac{d}{dt}$ in rotating frame by $\frac{d'}{dt}$ then

$$\frac{d\vec{A}}{dt} = \frac{d'\vec{A}}{dt} + \vec{\omega} \times \vec{A} \quad \text{(where } \vec{A} \text{ is any quantity)}$$

$$\therefore \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \left(\frac{d'}{dt} + \vec{\omega} \times\right)\left(\frac{d'\vec{r}}{dt} + \vec{\omega} \times \vec{r}\right)$$

$$= \frac{d'^2\vec{r}}{dt^2} + 2\vec{\omega} \times \frac{d'\vec{r}}{dt} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

we have neglected the term due to nonuniform rotation. $\left(\frac{d'\vec{\omega}}{dt} \times \vec{r}\right)$

The term $2m\vec{\omega} \times \frac{d'\vec{r}}{dt}$ is called Coriolis force &

$2\vec{\omega} \times \frac{d'\vec{r}}{dt}$ is called Corilis acceleration.

The equation of motion $m\frac{d'^2\vec{r}}{dt^2} = \vec{F} - 2m\left(\vec{\omega} \times \frac{d'\vec{r}}{dt}\right) - m\vec{\omega} \times (\vec{\omega} \times \vec{r})$

(in this equation of motion the term $m\frac{d'\vec{\omega}}{dt} \times \vec{r}$ is neglected as it is a term due to nonuniform rotation)

The term $m\vec{\omega} \times (\vec{\omega} \times \vec{r})$ is called centrifugal force.

Thus in a frame rotating relative to an inertial frame additional forces like CORIOLIS FORCE AND CENTRIFUGAL FORCE appear.

Indeed mechanics, electromagnetism & all other phenomena make no distinction between two frames in uniform relative velocity.